



# **Confirmation Theory**

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## Please interrupt

P1. If H, then E

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P2. *H* 

PI. If *H*, then *E* P2. *H* 

• In an *ampliative* inference, the truth of the premises *doesn't* guarantee the truth of the conclusion.

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$$\begin{array}{ccc} H \longrightarrow E & H \longrightarrow E \\ \hline H & & \neg H \\ \hline E & & \neg E \end{array}$$

• Inductive Logic is the study of which inductive inferences are good *qua* inductive inferences and which are bad *qua* inductive inferences.

• Confirmation Theory is the study of which inductive inferences are good *qua* inductive inferences and which are bad *qua* inductive inferences.

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- Question 2: Why should we think that good inductive inferences will lead us to truth?
- ▷ David Hume: there is no non-circular answer to Question 2.

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## Confirmation & Disconfirmation

- Sometimes, a piece of evidence, *E*, gives reason to believe a hypothesis, *H*.
  - When this is so, say that *E confirms H*.
- Other times, a piece of evidence, *E*, gives reason to *dis*believe a hypothesis, *H*.
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• Just because we have some evidence, *E*, which confirms *H*, this doesn't mean that we should think *H* is true.

#### ▷ Confirmation comes in degrees.

- ▷ E could give a very strong reason to believe that H, or it could give a rather weak reason to believe that H.
- ▷ If *E* confirms *H*, but only very weakly, then it could be that we shouldn't believe *H*.

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## Confirmation is Degreed



• Just because we have some evidence, *E*, which confirms *H*, this doesn't mean that we should think *H* is true.

- Let *H* be the hypothesis that John robbed the bank, and let *E* be the evidence that 100 eyewitnesses who know John personally identified John as the bank robber.
- ▷ *E* strongly confirms *H*. But just because you have the evidence *E*, this doesn't mean that you should believe *H*.
- ▷ *E* needn't be your *total* evidence.
- You could additionally have the evidence that John has an identical twin brother and that John has a rock-solid alibi.

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- Deductive inference is monotonic (or indefeasible)
  - If *P deductively entails C*, then *P*&*Q* deductively entails *C* as well.
- Inductive inference is non-monotonic (or defeasible)
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- A *qualitative* account of confirmation.
  - For any *H*, *E*: does *E* confirm *H*?
- A *quantitative* measure of confirmation.
  - For any H, E: to what degree does E confirm H?
- We'd like our theory of confirmation to be *formal* and *intersubjective*.
  - Formal: we can say whether E confirms H by looking only at syntax, or logical form.
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# You Can't Always Get What You Want

• A promising first thought: deductive consequences of a hypothesis confirm it.

**Entailments** Confirm

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• Another promising thought: confirmation transmits through deduction.

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If *E* confirms *H*, then *E* confirms anything which *H* entails.

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#### **Entailments Confirm**

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#### **Consequence Condition**

If E confirms H, then E confirms anything which H entails.

• If we accept both Entailments Confirm and the Consequence Condition, then we must say that every proposition confirms every other proposition.









#### **Entailments Confirm**

If *H* entails *E*, then *E* confirms *H*.

#### **Consequence Condition**

If E confirms H, then E confirms anything which H entails.

• Perhaps we should weaken these principles.

#### Laws are Confirmed by Their Instances

A law statement of the form "All *F*s are *G*s" is confirmed by an *F G*.

#### **Consequence Condition**

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#### Laws are Confirmed by Their Instances

A law statement of the form "All Fs are Gs" is confirmed by an F G.

#### **Equivalence Condition**

If E confirms H, then E confirms anything equivalent to H.

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#### Laws are Confirmed by Their Instances

A law statement of the form "All *F*s are *G*s" is confirmed by an *F G*.

#### **Equivalence Condition**

If E confirms H, then E confirms anything equivalent to H.

• New Problem: *nearly everything* confirms any given law statement.

- 1. "All ravens are black" is equivalent to "All non-black things are non-ravens".
- 2. By Laws are Confirmed by Their Instances, a green leaf (which is both a non-black thing and a non-raven) confirms the hypothesis that all non-black things are non-ravens.
- 3. By 1, 2, and the Equivalence Condition, a green leaf confirms the hypothesis that all ravens are black.

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The first observed emerald is green The second observed emerald is green : The *n*th observed emerald is green All unobserved emeralds are green • Say that a thing is grue iff it has been observed before 2018 and is green or has not been observed before 2018 and is blue.

### Goodman's Impossibility Result



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The *n*th observed emerald is green

All unobserved emeralds are green

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The first observed emerald is grue The second observed emerald is grue

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The *n*th observed emerald is grue

All unobserved emeralds are blue

- If "All unobserved emeralds are green" is confirmed by the observation of *n* green emeralds, then so too is the hypothesis that "All unobserved emeralds are blue".
- A purely *formal* theory of confirmation cannot distinguish induction from counter-induction.
- So a theory of induction must go beyond logical form.

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#### What We Want from a Theory of Confirmation

- A *qualitative* account of confirmation.
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## Confirmation & Probability

# Confirmation & Probability

• A *probability function*, Pr, is any function from a set of propositions,  $\mathscr{P}$ , to the unit interval, [0, I]

 $\mathsf{Pr}:\mathscr{P}\to[\mathsf{O},\mathrm{I}]$ 

#### which also has the following properties:

Ax1. If the proposition  $\top$  is necessarily true, then  $\Pr(\top) = I$ . Ax2. If the propositions *A* and *B* are inconsistent, then  $\Pr(A \lor B) = \Pr(A) + \Pr(B)$ . • A *probability function*, Pr, is any function from a set of propositions,  $\mathscr{P}$ , to the unit interval, [0, I]

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which also has the following properties: AxI. If the proposition T is necessarily true, then Pr(T) = I. Ax2. If the propositions A and B are inconsistent, then  $Pr(A \lor B) = Pr(A) + Pr(B)$ . • A *probability function*, Pr, is any function from a set of propositions,  $\mathscr{P}$ , to the unit interval, [0, I]

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Α	B	С	Pr
Τ	T	T	6/16
Τ	T	F	2/16
Τ	F	T	2/16
Τ	F	F	1/16
F	T	T	2/16
F	T	F	1/16
F	F	T	1/16
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• We introduce the following *definition*:

$$\Pr(A \mid B) \stackrel{\text{def}}{=} \frac{\Pr(A \& B)}{\Pr(B)}$$
, if defined

#### **Conditional Probability**



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## **Conditional Probability**



• We may say that the propositions *A* and *B* are *independent* (according to Pr) if and only if

$$\Pr(A \& B) = \Pr(A) \cdot \Pr(B)$$

• We may say that the propositions *A* and *B* are *independent* (according to Pr) if and only if

$$\frac{\Pr(A \& B)}{\Pr(B)} = \Pr(A)$$

• We may say that the propositions *A* and *B* are *independent* (according to Pr) if and only if

 $\Pr(A \mid B) = \Pr(A)$ 

## Confirmation & Probability

From Probability to Confirmation

- Given a probability function Pr, we may construct a *confirmation measure*  $\mathfrak{C}$ ,
  - $\mathfrak{C}(H, E)$  gives the degree to which the evidence *E* confirms the hypothesis *H*.
- One popular confirmation measure:

 $\mathfrak{D}(H,E) = \Pr(H \mid E) - \Pr(H)$ 

$$\Re(H, E) = \log\left(\frac{\Pr(H \mid E)}{\Pr(H)}\right)$$
$$\Re(H, E) = \log\left(\frac{\Pr(E \mid H)}{\Pr(E \mid \neg H)}\right)$$

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$$\mathcal{L}(H, E) = \log\left(\frac{\Pr(E \mid H)}{\Pr(E \mid \neg H)}\right)$$

- All of these measures will agree about the following:
  - If  $\Pr(H | E) > \Pr(H)$ , then *E* confirms *H*
  - If Pr(H | E) < Pr(H), then *E* disconfirms *H*
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### Probability & Confirmation

• It is a consequence of the definition of conditional probability that:

$$\Pr(H \mid E) = \frac{\Pr(E \mid H)}{\Pr(E)} \cdot \Pr(H)$$

• So, we may say: *E* confirms *H* if and only if

 $\Pr(H \mid E) > \Pr(H)$ 

• That is: *E* confirms *H* if and only if *H* did a good job *predicting E*.
• It is a consequence of the definition of conditional probability that:

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- *E* confirms *H* if and only if *H* did a good job predicting *E*.
- What do we mean by 'good job'?
  - In order to do a good job predicting *E*, *H* doesn't have to make *E* likely.
  - Also, in order to do a good job predicting *E*, it is not enough for *H* to make *E* likely.
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#### • The suspect confesses.

- This is *very* unlikely, given that the suspect is guilty. Guilty suspects almost never confess.
- However, it is even *less* likely that the suspect confesses, given that the suspect is innocent.
- So, a confession confirms the hypothesis that the suspect was guilty, even though a confession was *very* unlikely given that the suspect was guilty.

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- Hypothesis, *H*: Bob wears a helmet on his bike ride into work.
- Evidence, *E*: Bob makes it into work without getting into an accident.
- Given *H*, *E* is *very* likely.
- However, *E* is even *more* likely given  $\neg H$ .
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# Break

# **Bayesian Confirmation Theory**

- Our theory of confirmation says nothing until we say more about the probability function Pr.
- The Bayesian thinks that Pr represents some hypothetical rational agent's *degrees of belief*, or *credences*.
  - If Pr(A) = 1, then the agent thinks that A is certainly true.
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- Suppose that, upon acquiring the total evidence *E*, you are disposed to adopt some new degrees of belief, Pr<sub>E</sub>.
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It is a requirement of rationality that, upon acquiring the total evidence *E*, you are disposed to adopt a new credence function Pr<sub>E</sub> which is your old credence function *conditionalized on E*. That is, for all *H*,

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• The Bayesian theory of confirmation says that *E* confirms *H* iff

 $\Pr_E(H) > \Pr(H)$ 

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### Justifying Bayesianism Pragmatically

- A pragmatic justification of probabilism: If your degrees of belief don't satisfy the axioms of probability, then you could be sold a combination of bets which is guaranteed to lose you money *come what may*.
- A pragmatic justification of conditionalization: If you stand to learn whether *E*, and you are disposed to revise your beliefs in any way other than conditionalization, then you could be reliably sold a series of bets which are guaranteed to lose you money no matter what.

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# Why the Bayesian Thinks You Can't Always Get What You Want

#### **Entailments Confirm**

If *H* entails *E*, then *E* confirms *H* 

#### **Consequence Condition**

If E confirms H, then E confirms anything which H entails.

• If we accept both Entailments Confirm and Consequence Condition, then every proposition confirms every other proposition.

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- We are playing poker, and I catch a glimpse of your cards. I see that you have a spade.
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#### Laws are Confirmed by Their Instances

A law statement of the form "All Fs are Gs" is confirmed by an F  ${\cal G}$ 

#### **Equivalence Condition**

If E confirms H, then E confirms anything equivalent to H

• If we accept both of these principles, then we must say that a green leaf confirms the hypothesis that "All ravens are black".

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• If H is equivalent to  $H^*$ , then  $Pr(H) = Pr(H^*)$  and  $Pr(H \& E) = Pr(H^* \& E)$ . So

$$\Pr(H^* \mid E) = \frac{\Pr(H^* \& E)}{\Pr(E)} = \frac{\Pr(H \& E)}{\Pr(E)} = \Pr(H \mid E)$$

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- E = a randomly selected thing is a non-black non-raven.
- As we saw, *E* will confirm *All* iff *All* makes *E* more likely than *Some*. But

$$\Pr(E \mid All) = I/4$$
 and  $\Pr(E \mid Some) = I/4$ 

- So the Universal hypothesis *All* is not confirmed by a non-black non-raven.
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### • *Green* = All emeralds are green

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## The Problem of the Priors

- If we want *Green* to have a higher posterior credence than *Grue*, then we must stipulate that *Green* has a higher *prior* credence than *Grue*.
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#### What We Want from a Theory of Confirmation

- A *qualitative* account of confirmation.
  - For any *H*, *E*: does *E* confirm *H*?
- A *quantitative* measure of confirmation.
  - For any *H*, *E*: to what *degree* does *E* confirm *H*?
- We'd like our theory of confirmation to be *formal* and *intersubjective*.
  - *Formal*: we can say whether *E* confirms *H* by looking only at syntax, or logical *form*.
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- Radical Subjectivism: All probabilistic priors are rationally permissible.
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