# Confirmation Theory

Pittsburgh Summer Program 1

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## 1 Confirmation & Disconfirmation

1. Sometimes, a piece of evidence, $E$, gives reason to believe a hypothesis, $H$. When this is so, say that $E$ *confirms* $H$.

2. Other times, a piece of evidence, $E$, gives reason to *dis*believe a hypothesis, $H$. When this is so, say that $E$ *disconfirms* $H$.

3. Just because we have some evidence, $E$, which confirms $H$, this doesn't mean that we should think $H$ is true.

   (a) Confirmation is a matter of degree. $E$ could confirm $H$ by giving a *slight* but not conclusive reason to believe $H$.

   (b) Evidence for $H$ could be defeated. We could have *some* evidence $E$ which confirms $H$ while having a *total* body of evidence which disconfirms $H$.

      i. While deductive inference is *monotonic* (or indefeasible),

         ▹ If $P$ deductively entails $C$, then $P \& Q$ also deductively entails $C$

         inductive inference is *non-monotonic* (or defeasible).

         ▹ If $E$ confirms $H$, it doesn't follow that $E \& F$ confirms $H$

4. What we want from a theory of confirmation:

   (a) A *qualitative* account of confirmation.

      i. For any $H, E$: does $E$ confirm $H$?

   (b) A *quantitative* measure of confirmation.

      i. For any $H, E$: to what *degree* does $E$ confirm $H$?

   (c) We'd like our theory of confirmation to be both *formal* and *intersubjective*.

      i. *Formal*: we can say whether $E$ confirms $H$ by looking only at the syntax, or logical form, of $E$ and $H$.

      ii. *Intersubjective*: we can all agree about whether $E$ confirms $H$.

## 2 You can't always get what you want

5. Hempel (1945a,b): any theory of confirmation which satisfies these two plausible principles will say that every proposition confirms every other proposition.

   Entailments Confirm (ec)
   If $H$ entails $E$, then $E$ confirms $H$.

   Consequence Condition (cc)
   If $E$ confirms $H$, then $E$ confirms anything which $H$ entails.

   (a) Take any two propositions $A$ and $B$.

   (b) $A \& B$ entails $A$. So, by ec, $A$ confirms $A\&B$.

   (c) $A$ confirms $A\&B$ (above) and $A\&B$ entails $B$. So, by cc, $A$ confirms $B$

6. Hempel: even if we weaken these principles like this,

   Laws are Confirmed by Their Instances
   A law statement of the form "All $F$s are $G$s" is confirmed by an $F$ $G$.

EQUIVALENCE CONDITION
If $E$ confirms $H$, then $E$ confirms anything which is equivalent to $H$.

*nearly everything* will end up confirming any universal law statement. Take, for a toy example, the law statement "All ravens are black".

(a) "All ravens are black" is equivalent to "All non-black things are non-ravens"

(b) By LAWS ARE CONFIRMED BY THEIR INSTANCES, a green leaf confirms the hypothesis that all non-black things are non-ravens

(c) By (a), (b), and EQUIVALENCE CONDITION, a green leaf confirms the hypothesis that all ravens are black.

7. GOODMAN (1955): in order to say whether "All $F$s are $G$s" is confirmed by an $F$ $G$, we must know something about what '$F$' and '$G$' *mean*.

(a) Say that a thing is grue iff it has been observed before 2018 and is green or has not been observed before 2018 and is blue.

(b) Then, there is no *syntactic, formal* difference between *this* inductive inference (which is a strong inductive inference):

The first observed emerald is green
The second observed emerald is green
⋮
The $n$th observed emerald is green
═══════════════════
All unobserved emeralds are green

and *this* inductive inference (which is a counter-inductive inference):

The first observed emerald is grue
The second observed emerald is grue
⋮
The $n$th observed emerald is grue
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All unobserved emeralds are grue

8. A purely formal theory of confirmation cannot distinguish induction from counterinduction. So a theory of confirmation must go beyond logical form.

## 3 CONFIRMATION & PROBABILITY

### 3.1 PROBABILITY

9. A *probability function*, Pr, is any function from a set of propositions, $\mathscr{P}$, to the unit interval, $[0,1]$

$$\Pr : \mathscr{P} \to [0,1]$$

which also has the following properties:

AX1. If the proposition $\top$ is necessarily true, then $\Pr(\top) = 1$.

AX2. If the propositions $A$ and $B$ are inconsistent, then $\Pr(A \vee B) = \Pr(A) + \Pr(B)$.

10. If Pr is a probability function, then we may represent it with a *muddy Venn diagram* or a *probabilistic truth-table*.

11. We introduce the following *definition*:

$$\Pr(A \mid B) \overset{\text{def}}{=} \frac{\Pr(A \,\&\, B)}{\Pr(B)}, \quad \text{if defined}$$

12. We may say that the propositions $A$ and $B$ are *independent* (according to Pr) if and only if

$$\Pr(A \& B) = \Pr(A) \cdot \Pr(B)$$

### 3.2 FROM PROBABILITY TO CONFIRMATION

Given a probability function Pr, we may construct a *confirmation measure* $\mathfrak{C}$,

(a) $\mathfrak{C}(H,E)$ gives the degree to which the evidence $E$ confirms the hypothesis $H$.

13. One popular confirmation measure:

$$\mathfrak{D}(H,E) = \Pr(H \mid E) - \Pr(H)$$

(a) There are other possibilities—*e.g.*,

$$\mathfrak{R}(H,E) = \log\left(\frac{\Pr(H \mid E)}{\Pr(H)}\right)$$

$$\mathfrak{L}(H,E) = \log\left(\frac{\Pr(E \mid H)}{\Pr(E \mid \neg H)}\right)$$

14. All of these measures will agree about the following:
    (a) If $\Pr(H \mid E) > \Pr(H)$, then $E$ confirms $H$
    (b) If $\Pr(H \mid E) < \Pr(H)$, then $E$ disconfirms $H$
    (c) If $\Pr(H \mid E) = \Pr(H)$, then $E$ neither confirms nor disconfirms $H$
15. It is a consequence of the definition of conditional probability that:

$$\Pr(H \mid E) = \frac{\Pr(E \mid H)}{\Pr(E)} \cdot \Pr(H)$$

16. So, we may say: $E$ confirms $H$ if and only if

$$\Pr(H \mid E) > \Pr(H)$$
$$\frac{\Pr(E \mid H)}{\Pr(E)} \cdot \Pr(H) > \Pr(H)$$
$$\Pr(E \mid H) > \Pr(E)$$

That is: $E$ confirms $H$ if and only if $H$ did a good job *predicting* $E$.

(a) In order to do 'a good job' predicting $E$, $H$ doesn't have to make $E$ likely.
(b) Also, in order to do a good job predicting $E$, it is not enough for $H$ to make $E$ likely.
(c) To do a good job predicting $E$, $H$ must make $E$ more likely than its negation, $\neg H$.

## 4 Bayesian Confirmation Theory

17. The Bayesian interprets Pr as providing the *degrees of belief*, or the *credences*, of some rational agent.
    (a) If $\Pr(A) = 1$, then the agent thinks that $A$ is certainly true.
    (b) If $\Pr(A) = 0$, then the agent thinks that $A$ is certainly false.
    (c) If $\Pr(A) = 1/2$, then the agent is as confident that $A$ is true as they are that $A$ is false.
18. The Bayesian then endorses the following norms of rationality:

Probabilism
It is a requirement of rationality that your degrees of belief Pr satisfy the axioms of probability.

Conditionalization
It is a requirement of rationality that, upon acquiring the total evidence $E$, you are disposed to adopt a new credence function $\Pr_E$ which is your old credence function *conditionalized on* $E$. That is, for all $H$,

$$\Pr_E(H) = \Pr(H \mid E) = \frac{\Pr(E \mid H)}{\Pr(E)} \cdot \Pr(H)$$

(a) Terminology:
    i. Pr is the agent's *prior* credence function.
    ii. $\Pr_E$ is the agent's *posterior* credence function.

19. The Bayesian theory of confirmation says that $E$ confirms $H$ iff

$$\Pr_E(H) > \Pr(H)$$

And $E$ disconfirms $H$ iff

$$\Pr_E(H) < \Pr(H)$$

20. A pragmatic justification of Bayesianism:
    (a) A pragmatic justification of probabilism: If your degrees of belief don't satisfy the axioms of probability, then you could be sold a combination of bets which is guaranteed to lose you money *come what may*. (Ramsey 1931)
    (b) A pragmatic justification of conditionalization: If you stand to learn whether $E$, and you are disposed to revise your beliefs in any way other than conditionalization, then you could be reliably sold a series of bets which are guaranteed to lose you money no matter what. (Teller 1976)

21. An Alethic justification of Bayesianism:

    (a) An alethic justification of probabilism: If your degrees of belief don't satisfy the axioms of probability, then there is some other degrees of belief you could adopt which is guaranteed to be *more accurate* than yours, no matter what. (Joyce 1998)

    (b) An alethic justification of conditionalization: If you stand to learn whether $E$, then the strategy of conditionalization has higher *expected accuracy* than any other strategy of belief-revision. (Greaves & Wallace 2006)

## 5 Why the Bayesian Thinks You Can't Always Get What You Want

22. Recall: these two principles jointly entail that every proposition confirms every other proposition:

    Entailments Confirm (ec)
    If $H$ entails $E$, then $E$ confirms $H$.

    Consequence Condition (cc)
    If $E$ confirms $H$, then $E$ confirms anything which $H$ entails.

23. Bayesian confirmation theory endorses Entailments Confirm, but rejects the Consequence Condition.

    (a) Against the Consequence Condition: If I see that you have a spade, this confirms that you have the ace of spades. And that you have the ace of spades entails that you have an ace. But that you have a spade does not confirm that you have an ace.

24. Recall: these two principles jointly entail that "All ravens are black" is entailed by a non-black non-raven.

    Laws are Confirmed by Their Instances
    A law statement of the form "All $F$s are $G$s" is confirmed by an $F$ $G$.

    Equivalence Condition
    If $E$ confirms $H$, then $E$ confirms anything which is equivalent to $H$.

25. Bayesian confirmation theory endorses Equivalence Condition, but rejects that Laws are Confirmed by their Instances.

    (a) A toy model: suppose that you are certain that there are 8 things in existence, and you split your credence equally between these two hypotheses about their properties:

**All**

| | Black | Non-Black |
|---|---|---|
| Raven | 4 | 0 |
| Non-Raven | 2 | 2 |

**Some**

| | Black | Non-Black |
|---|---|---|
| Raven | 2 | 2 |
| Non-Raven | 2 | 2 |

        i. You get the evidence $E$ = a randomly selected thing is a non-black non-raven.

        ii. As we saw, according to the Bayesian theory of confirmation, $E$ will confirm *All* iff *All* makes $E$ more likely than *Some* does. But

$$\Pr(E \mid \mathit{All}) = 1/4 \qquad \text{and} \qquad \Pr(E \mid \mathit{Some}) = 1/4$$

        iii. So the Universal hypothesis *All* is not confirmed by a non-black non-raven.

        iv. So Laws are Confirmed by Their Instances is false.

    (b) Contrast this with a case where you get the evidence $E^*$ = a randomly selected thing is a black raven.

        i. As we saw, according to the Bayesian theory of confirmation, $E^*$ will confirm *All* iff *All* makes $E$ more likely than *Some* does. And

$$\Pr(E^* \mid \mathit{All}) = 1/2 \qquad \text{and} \qquad \Pr(E^* \mid \mathit{Some}) = 1/4$$

        ii. So a black raven confirms *All*, even though a non-black non-raven does not.

26. Recall: a non-formal theory of confirmation cannot distinguish the Green hypothesis from the Grue hypothesis.

    (a) Notation:

        i. *Green* = All emeralds are green

        ii. *Grue* = All emeralds are grue

iii. $E$ = All observed emeralds are green/grue

(b) It turns out that the Bayesian can only say that *Green* is more likely than *Grue*, given the evidence $E$, if *Green started out* more likely than *Grue* in the prior. For

$$\frac{\Pr(Green \mid E)}{\Pr(Grue \mid E)} = \frac{\frac{\Pr(E|Green)}{\Pr(E)} \cdot \Pr(Green)}{\frac{\Pr(E|Grue)}{\Pr(E)} \cdot \Pr(Grue)} = \frac{\Pr(E \mid Green) \cdot \Pr(Green)}{\Pr(E \mid Grue) \cdot \Pr(Grue)} = \frac{\Pr(Green)}{\Pr(Grue)}$$

## 6 The Problem of the Priors

27. As the case of *Green* and *Grue* demonstrates, the probabilities assigned by the priors end up doing a lot of the heavy lifting in Bayesian confirmation theory.

28. The 'problem of the priors' is the problem of specifying which prior credence functions are rational.

29. Four different kinds of answers to the problem of the priors:

    (a) Radical Subjectivism: All probabilistic priors are rationally permissible.

    (b) Only slightly less radical subjectivism: Any probabilistic prior is rationally permissible so long as it satisfies a PROBABILITY COORDINATION PRINCIPLE like

    $$\text{if } H \text{ gives } E \text{ an objective chance of } x\text{, then } \Pr(E \mid H) = x \qquad \text{(PCP)}$$

    (c) Moderate Subjectivism: There is a limited range of rationally permissible priors.

    (d) Objectivism: There is only one rational prior.

30. The *Principle of Indifference* undergirds one historically prominent version of Objectivism.

    THE PRINCIPLE OF INDIFFERENCE
    In the absence of evidence, assume a *uniform* credence distribution.

31. It is commonly thought, however, that the Principle of Indifference is contradictory.

    (a) Consider the following case:

    CROSS-COUNTRY DRIVE (V1)
    I drove 2100 miles from Pittsburgh to L.A. The trip took somewhere between 30 and 42 hours. If you know nothing else about my trip, what is the rational credence to have that it took between 30 and 35 hours?

    Applying the principle of indifference, we suppose that the probability it took between $h$ hours and $h+1$ hours is the same, for all $h$ between 30 and 41. So, if $t$ is the time it took, we conclude that

    $$\Pr(30 \le t \le 35) = \frac{5}{12}$$

    (b) Then consider this case:

    CROSS-COUNTRY DRIVE (V2)
    I drove 2100 miles from Pittsburgh to L.A. My average velocity was somewhere between 50 and 70 mph. What is the rational credence to have that the average velocity was between 60 and 70 mph?

    Applying the principle of indifference, we suppose that the probability of the average velocity being between $m$ and $m+1$ miles per hour is the same, for all $m$ between 50 and 69. So, if $v$ is the average velocity, we conclude that

    $$\Pr(60 \le v \le 70) = \frac{1}{2}$$

    (c) The punchline: CROSS-COUNTRY DRIVE (V1) and CROSS-COUNTRY DRIVE (V2) are *precisely the same case*, just described differently. And $30 \le t \le 35$ if and only if $60 \le v \le 70$. So PROBABILISM says that they must receive the same probability. The principle of indifference assigns them different probabilities. So the principle of indifference contradicts itself.

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