Two-Dimensional *De Se* Chance Deference

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1 Lewis's Principle of Chance Deference

1. Lewis (1980) defended the following principle of chance deference:[1,2]

   Lewis's Principle of Chance Deference
   For any thought 'p', any number n%, and any time t,

   $$C(p \mid \mathcal{C}h_t(p) = n\%) \overset{!}{=} n\% \qquad \text{(LCD)}$$

   (so long as you lack any time t inadmissible information)

   (a) As I use the term, *thoughts* are the arguments of your credence function

   (b) For Lewis (1980), information is time t *inadmissible* iff it is *about* times after t

2. I'll argue here that Lewis's principle LCD faces two kinds of difficulties:

   (a) In the first place, it faces difficulties with *a priori* knowable contingencies. (This difficulty has been discussed by Hawthorne & Lasonen-Aarnio (2009), Salmón (2019), and Nolan (2016), among others.)

   (b) In the second place, it faces difficulties in cases where you've lost track of the time. (To my knowledge, I am the first to note these difficulties.)

3. Problem #1: suppose that we are about to flip a coin, but before we do so, we introduce the name 'Uppy' by saying: "Let's call whichever side of the coin actually lands up 'Uppy'."

   (a) Let 'u' be the thought that the coin lands with Uppy facing up.

   (b) Then, if we set p equal to u and we set n equal to 50, LCD tells us that

   $$C(u \mid \mathcal{C}h_t(u) = 50\%) \overset{!}{=} 50\%$$

   (c) But you know for sure that the chance of u is 50%. Uppy is either heads or tails. If Uppy is heads, then the chance of the coin landing on Uppy is 50%. And if Uppy is tails, then the chance of the coin landing on Uppy is 50%. So, either way, the chance of the coin landing on Uppy is 50%. So $C(\mathcal{C}h_t(u) = 50\%) = 100\%$.

   (d) If $C(q) = 100\%$, then $C(p \mid q) = C(p)$. So LCD implies that your credence in 'u' should be 50%.

   (e) But it is *a priori* knowable that the coin lands on Uppy (so long as it lands on anything at all). So your credence in 'u' should be close to 100%, and not down around 50%.

4. Problem #2: suppose that you don't know whether it is Tuesday or Wednesday. You think it's 50% likely to be Tuesday and that it's 50% likely to be Wednesday. But you know for sure that *today*'s chance of Mudskipper winning ('m') is 75%, and *yesterday*'s chance of Mudskipper winning was 25%.

   (a) If we set p equal to m, t equal to *tues*, and n equal to 25 and 75, respectively, LCD tells us that

   $$C(m \mid \mathcal{C}h_{tues}(m) = 25\%) \overset{!}{=} 25\%$$

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[1] LCD isn't the same as Lewis's *Principal Principle*, though it follows from the Principal Principle given the updating rule of conditionalization, which Lewis accepted (see his 1999, e.g.).

[2] Notation: I place an exclamation mark above an equals sign to say that the equality *should* hold, not that it *does* hold.

$$\text{and} \qquad C(m \mid Ch_{tues}(m) = 75\%) \overset{!}{=} 75\%$$

Since you know for sure that $Ch_{tues}(m) = 25\%$ iff it is Wednesday, and you know for sure that $Ch_{tues}(m) = 75\%$ iff it is Tuesday, this implies that

$$C(m \mid weds) \overset{!}{=} 25\%$$
$$\text{and} \qquad C(m \mid tues) \overset{!}{=} 75\%$$

(b) Since you are 50% sure that it is Tuesday and 50% sure that it is Wednesday, this implies that your credence in 'm' should be 50%,[3]

$$\begin{aligned} C(m) &\overset{!}{=} C(m \mid tues) \cdot C(tues) \; + \; C(m \mid weds) \cdot C(weds) \\ &= 75\% \cdot 50\% \; + \; 25\% \cdot 50\% \\ &= 50\% \end{aligned}$$

(c) But this is implausible. You know for sure that *today*'s chance of Mudskipper winning is 75%. Given that, your credence that Mudskipper wins should be 75%, and not 50%.

2 A Two-Dimensional, *De Se* Principle of Chance Deference

5. Principle of chance deference are just one species of a broader genus of principles of *expert* deference. In general, a principle of expert deference says: given that *the expert* is n% confident in 'p', you should be n% confident in 'p', too.

6. But principles like these face difficulties when it comes to *de se* thoughts (thoughts which are in part about who you are and where you are located in space and time).

   (a) For instance, let the relevant expert be my doctor, and set 'p' equal to the *de se* thought 'I am sick'. Then, the principle will tell me:

   > Given that my doctor is n% confident in 'I am sick', I should be n% confident in 'I am sick'.

   But this is terrible advice. My doctor's thought 'I am sick' has the truth-conditional content that *she* is sick, not that *I* am sick.

   (b) I shouldn't defer to my doctor by setting my credence in 'I am sick' equal to her credence in *that same de se thought*. Instead, I should defer to her by setting my credence in 'I am sick' equal to her credence in some appropriately chosen *de dicto surrogate* of that *de se* thought. In this case, the appropriate surrogate is 'Dmitri is sick'.

To introduce the surrogate I think you should use *in general* when deferring to an expert, let me introduce the notion of a *location*.

2.1 Locations and *De-Dicto* Surrogates

7. A *purely de se* thought is a thought which only says something about who you are, or when and where you are located in time and space. And it does not say anything stronger.

   (a) E.g., 'Today is Monday' and 'I am Beyoncé' are both *purely de se* thoughts

8. A *location* is a thought which is strong enough to settle the truth-value of *all* of your purely *de se* thoughts—and no more. (It can of course settle the truth-values of thoughts which logically follow from your purely *de se* thoughts, but it can't settle the truth-values of any thoughts other than those.)

   (a) In other words: a *location* says who you are, where you are, and what time it is in as rich a detail as your thoughts will permit (and it doesn't say anything more than this).

   (b) As a notational convention, I'll use lowercase Greek letters for locations.

9. Now, take any thought, 'p', and any location, 'λ'. Then, the *de dicto* λ-*surrogate* of 'p'—which I will write 'p_λ'—is a thought which is true so long as 'p' is true when entertained at the location λ.

   (a) That is: 'p_λ' says that the thought 'p' expresses a truth when it is entertained at λ.

[3] So long, that is, as your credence function is a probability (I'll take for granted here that it should be).

(b) For instance, if 'δ' is my location, and 's' is the *de se* thought 'I am sick', then the *de dicto* δ-surrogate of 's'—'s_δ'—says that 'I am sick' is true when entertained at Dmitri's location. That is: 's_δ' says that Dmitri is sick.

10. Introducing *de dicto* surrogates is enough to solve our first problem for LCD. Recall: 'u' says that the coin lands on Uppy, where 'Uppy' is a name for whichever side the coin actually lands on. Let 'λ' be your (known) location. Then, the proposal is that you shouldn't defer to chance about *whether the coin lands on Uppy*. Instead, you should defer to chance about whether *your thought* 'u' expresses a truth. That is: you should satisfy:

$$C(u \mid \mathcal{C}h_t = Ch_t) \stackrel{!}{=} Ch_t(u_\lambda)$$

Even though there's only a 50% chance that the coin will land on Uppy, there is a *100%* chance that your thought 'the coin lands on Uppy' will express a truth. If the coin lands heads, then your thought 'the coin lands on Uppy' will say that the coin lands on heads, and this will be true. On the other hand, if the coin lands tails, then your thought 'the coin lands on Uppy' will say that the coin lands on tails, and this will be true. So your thought will be true no matter how the coin lands. So the principle will say that

$$C(u \mid \mathcal{C}h_t = Ch_t) \stackrel{!}{=} 100\%$$

And since it will say this for *every* potential chance function Ch_t, the principle will imply that your (unconditional) credence in 'u' should be 100%.

11. This works well so long as you know for sure what your location is. But what if you are uncertain about your location? Suppose, for instance, that I want to defer to my doctor about whether I'm sick, but I don't know whether I am Dmitri or Beyoncé.

(a) Then, I think we should say this: given that I am Dmitri and my doctor is n% sure that Dmitri is sick, I should be n% sure that I am sick. And, given that I am Beyoncé and my doctor is n% confident that Beyoncé is sick, I should be n% sure that I am sick.

(b) That is, 'δ' is Dmitri's location, 'β' is Beyoncé's location, '$\mathcal{D}$' is the definite description 'my doctor's credence function', and 'D' is any probability function, then my credence function should satisfy:

$$C(s \mid \mathcal{D} = D \wedge \delta) \stackrel{!}{=} D(s_\delta)$$

$$\text{and} \qquad C(s \mid \mathcal{D} = D \wedge \beta) \stackrel{!}{=} D(s_\beta)$$

12. More generally, I think you should defer to an expert, $\mathcal{E}$, as described below:

Two-Dimensional *De Se* Deference
Given that the expert $\mathcal{E}$'s probability function is E, and given that you are located at λ, your credence in 'p' should be E's probability in the *de dicto* λ-surrogate of 'p', 'p_λ'.

$$C(p \mid \mathcal{E} = E \wedge \lambda) = E(p_\lambda)$$

In a slogan: you should defer to the expert about whether your thoughts are true, given the location at which you are entertaining them.

2.2 Chance Deference

13. In the case of chance, then, we should say this:

Two-Dimensional *De Se* Chance Deference
So long as you lack any time t inadmissible information, your credence in 'p', given that the time t objective chance function is Ch_t, and given that you are located at λ, should be equal to $Ch_t(p_\lambda)$.

$$C(p \mid \mathcal{C}h_t = Ch_t \wedge \lambda) = Ch_t(p_\lambda)$$

In a slogan: you should defer to chance about whether your thoughts are true, given the location at which you are entertaining them.

14. The principle above only applies in cases where you lack inadmissible information. However, if we assume ur-prior conditionalization, it tells us exactly what your credences should be, even if you have inadmissible information.

To see why, suppose that your total evidence is 'e', which may or may not be inadmissible. Then, ur-prior conditionalization says that there's some ur-prior credence function C_0 such that your current credence ought to be C_0 conditioned on 'e'. So:

$$C(p \mid \mathcal{C}h_t = Ch_t \wedge \lambda) \stackrel{!}{=} C_0(p \mid \mathcal{C}h_t = Ch_t \wedge \lambda \wedge e)$$

$$= \frac{C_0(p \wedge e \mid \mathcal{C}h_t = Ch_t \wedge \lambda)}{C_0(e \mid \mathcal{C}h_t = Ch_t \wedge \lambda)}$$

Now, we may apply the principle of chance deference to both the numerator and the denominator of the fraction above. After all, the ur-prior credence function C_0 doesn't have any inadmissible evidence—it doesn't have any evidence at all! So:

$$C(p \mid \mathcal{C}h_t = Ch_t \wedge \lambda) \stackrel{!}{=} \frac{Ch_t(p_\lambda \wedge e_\lambda)}{Ch_t(e_\lambda)} = Ch_t(p_\lambda \mid e_\lambda)$$

(a) This implies that, if your total evidence is time t admissible, then, for any potential time t chance function Ch_t, $Ch_t(e_\lambda) = 100\%$.

(b) I propose strengthening this necessary condition on admissibility into a criterion of inadmissibility. That is, I propose:

> Inadmissible Information
> e is *inadmissible* for the time t chances iff, for some potential location λ and some potential time t chance function Ch_t,
>
> $$Ch_t(e_\lambda) < 100\%$$

▹ Here, a location is a *potential* location iff your credence that it is your location is greater than 0%. Likewise, a chance function Ch_t is a *potential* time t chance function iff your credence in $\mathcal{C}h_t = Ch_t$ is greater than 0%.

In a slogan: e is inadmissible just in case it might be news to the objective chances.

15. Given this criterion for inadmissibility, we can give a fully general principle of chance deference, which applies even in cases where you have inadmissible information:

> Two-Dimensional *De Se* Chance Deference (v2)
> If 'e' is your time t inadmissible information, then your credence in 'p', given that the time t objective chance function is Ch_t and given that you are located at λ, should be equal to $Ch_t(p_\lambda \mid e_\lambda)$.
>
> $$C(p \mid \mathcal{C}h_t = Ch_t \wedge \lambda) = Ch_t(p_\lambda \mid e_\lambda) \qquad \text{(CD)}$$

16. This principle allows us to solve Problem #2 (Losing Track of the Time).

(a) Recall, in the problem case, you are 50% sure that it is Tuesday, 50% sure that it is Wednesday, and you know for sure that *today* the chance of Mudskipper winning ('m') is 75%, and that *yesterday* the chance of m was 25%.

(b) Let 'τ' be the location 'it is Tuesday' and let 'ω' be the location 'it is Wednesday'

(c) Then, notice that the information '$\mathcal{C}h_{today}(m) = 75\%$' is inadmissible for the Tuesday chances. For ω is a potential location, and the *de dicto* ω-surrogate of '$\mathcal{C}h_{today}(m) = 75\%$' (namely: the *wednesday* chance of m is 75%) might be news to the Tuesday chances (if it's Wednesday, then the Tuesday chances don't know for sure that the Wednesday chance of 'm' is 75%).

(d) There are two relevant kinds of potential Tuesday chance functions: those according to which the chance of m is 75%, and those according to which the chance of m is 25%. Take an arbitrary function of the first kind and call it '$Ch^{75\%}$'. Take an arbitrary function of the second kind and call it '$Ch^{25\%}$'. You know for sure that $\mathcal{C}h_{tues} = Ch^{75\%}$ iff it is Tuesday, and you know for sure that $\mathcal{C}h_{tues} = Ch^{25\%}$ iff it is Wednesday. Then, CD implies that:

$$\begin{aligned} C(m \mid \mathcal{C}h_{tues} = Ch^{75\%} \wedge \tau) &\stackrel{!}{=} Ch^{75\%}(m \mid \mathcal{C}h_{today}(m) = 75\%_\tau) \\ &= Ch^{75\%}(m \mid \mathcal{C}h_{tues}(m) = 75\%) \\ \text{and} \quad C(m \mid \mathcal{C}h_{tues} = Ch^{25\%} \wedge \omega) &\stackrel{!}{=} Ch^{25\%}(m \mid \mathcal{C}h_{today}(m) = 75\%_\omega) \\ &= Ch^{25\%}(m \mid \mathcal{C}h_{weds}(m) = 75\%) \end{aligned}$$

(e) Assuming that the chance function knows its own values, $Ch^{75\%}(\mathcal{C}h_{tues}(m) = 75\%) = 100\%$, so the first constraint above implies that

$$C(m \mid \mathcal{C}h_{tues} = Ch^{75\%} \wedge \tau) \stackrel{!}{=} Ch^{75\%}(m) = 75\%$$

And, assuming that the objective chances satisfy the principle of reflection,[4] the second constraint above implies that

$$C(m \mid Ch_{tues} = Ch^{25\%} \wedge \omega) \overset{!}{=} 75\%$$

Since this will apply to *any* functions $Ch^{75\%}$ and $Ch^{25\%}$, conglomerability implies that your unconditional credence in 'm' should be 75%, and Problem #2 is resolved.

3 Sleeping Beauty

17. The principle of chance deference I've defended here has a surprising consequence for Elga (2000)'s *Sleeping Beauty* puzzle.

18. In this puzzle, we suppose that, on Sunday, you will be put to sleep with a powerful sedative and awoken on Monday morning. On Monday evening, you will be put back to sleep and a fair coin will be flipped. If this coin lands heads, then you will be kept asleep throughout Tuesday, and you not be awoken again until Wednesday. If, on the other hand, the coin lands tails, then your memories of Monday will be erased, and you will be awoken again on Tuesday. Also, just by the way: you are beautiful.

19. When you awake on Monday morning, you will know for sure that, if it is Tuesday, then the coin flip on Monday landed tails. However, you won't know for sure whether it is Monday or Tuesday. (For all you know for sure, it is Tuesday and your memories of being awoken on Monday have been erased.) The central debate over *Sleeping Beauty* is how confident you should be that Monday's coin flip lands heads ('h')

    - ▹ So-called *thirders* say that your credence in 'h' should be 1/3rd. They advocate the credence distribution shown in figure 1a.
    - ▹ So-called *halfers* are unhappy with this distribution, in part because it means that your credence in 'h' departs from the known *chance* of 'h'. They say that your credence in 'h' should be 1/2. They advocate the credence distribution shown in figure 1b.

[4] The principle of reflection, applied to the objective chances, says that, for any times t, t^* such that $t < t^*$, $Ch_t(p \mid Ch_{t^*}(p) = x) = x$. *Cf.* van Fraassen (1984)

	Monday	*Tuesday*
Heads	*1/3*	
Tails	*1/3*	*1/3*

(a)

	Monday	*Tuesday*
Heads	*1/2*	
Tails	*1/4*	*1/4*

(b)

Figure 1: The *thirder* thinks you should have the credence distribution in figure 1a, whereas the *halfer* thinks you should have the credence distribution in figure 1b.

20. Let's use 'μ' and 'τ' for the locations 'It is Monday' and 'It is Tuesday', respectively. Let 'Ch' be an arbitrary objective chance function such that $Ch(h) = 50\%$. And let 'a' be the thought 'I am awake'.

    - ▹ Importantly, 'a' is information you have when you awake on Monday—this is the information which allows you to rule out that it is Tuesday and Monday's flip landed heads.
    - ▹ Moreover, this information is *inadmissible* for the Monday chances, since τ is a potential location, and 'a_τ' ('I am awake on Tuesday') is news to the Monday chances. (Of course, 'a_μ' is *not* news to the Monday chances—the Monday chances know that you are awake on Monday.)

21. Then, CD implies that, for any potential chance function Ch,

    $$C(h \mid Ch_{mon} = Ch \wedge \mu) \overset{!}{=} Ch(h_\mu \mid a_\mu)$$

    But the Monday chances are already certain that a_μ, and the *de dicto* μ-surrogate of 'h_μ' is just 'h', so this reduces to

    $$C(h \mid Ch_{mon} = Ch \wedge \mu) \overset{!}{=} Ch(h) = 50\%$$

    Moreover, since this will hold for *any* potential Monday chance function Ch, conglomerability implies that

    $$C(h \mid \mu) \overset{!}{=} 50\%$$

This is a powerful constraint. It is incompatible with the halfer's distribution. (Though it is compatible with the thirder's.)

22. So, if we accept the principle of chance deference CD, then it is the *thirder*, and not the halfer, who properly defers to the known chances.

    (a) It's true that the thirder's credence in '*h*' does not *equal* the known chance of '*h*'. But, if we accept Inadmissibility, then the thirder has an excuse: their credence in '*h*' departs from the known chance of heads because they have the *inadmissible* information that they are awake. This information isn't *about* times after Monday. But, nonetheless, it is information which might be news to the Monday chances. And, given that they have this inadmissible information, they are correctly showing deference to the objective chances.

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