

Introduction to Predicate Logic

PHIL 500

$\forall x(Fx \rightarrow Gx), Fa \therefore Ga$

The Need for Predicate Logic

Translation into PL

Symbolization Keys

Important Statement Forms

The Need for Predicate Logic

The Need for PL

Everyone who has a dog is happy

Obama has a dog

\therefore Obama is happy

The Need for PL

Everyone who has a dog is happy

Obama has a dog

\therefore Obama is happy

▸ This argument is valid.

The Need for PL

Everyone who has a dog is happy

Obama has a dog

\therefore Obama is happy

- ▶ This argument is valid.
- ▶ But it isn't an entailment—so SL isn't able to tell us that it is valid.

The Need for PL

E

Obama has a dog

\therefore Obama is happy

- ▶ This argument is valid.
- ▶ But it isn't an entailment—so SL isn't able to tell us that it is valid.

The Need for PL

E

O

∴ Obama is happy

- ▶ This argument is valid.
- ▶ But it isn't an entailment—so SL isn't able to tell us that it is valid.

The Need for PL

E

O

$\therefore H$

- ▶ This argument is valid.
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The Need for PL

$E [T]$

O

$\therefore H$

- ▶ This argument is valid.
- ▶ But it isn't an entailment—so SL isn't able to tell us that it is valid.

The Need for PL

$E [T]$

$O [T]$

$\therefore H$

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The Need for PL

$E [T]$

$O [T]$

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- We'll have *names*, like

o : Obama

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- We'll have *names*, like

o : Obama

- We'll have *predicates*, like

D _____ : _____ has a dog

H _____ : _____ is happy

The Need for PL

- Our solution: to represent more of the structure of these statements
- We'll have *names*, like

o : Obama

- We'll have *predicates*, like

D _____ : _____ has a dog

H _____ : _____ is happy

- ▷ Putting them together will give *statements* like

Do : Obama has a dog

Ho : Obama is happy

The Need for PL

- Finally, we'll have two additional symbols, known as *quantifiers*:

$\forall x$ _____ : Everything is _____

$\exists x$ _____ : Something is _____

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- ▶ '∀' is an upside-down 'A'—it stands for '*all*'.
- ▶ '∃' is a backwards 'E'—it stands for '*exists*'.

The Need for PL

- Finally, we'll have two additional symbols, known as *quantifiers*:

$\forall x$ _____ : Everything is _____

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- ▶ ' \forall ' is an upside-down 'A'—it stands for '*all*'.
- ▶ ' \exists ' is a backwards 'E'—it stands for '*exists*'.
- ▶ ' x ' is a *variable*—we'll come back to this.

The Need for PL

Everyone who has a dog is happy

Obama has a dog

\therefore Obama is happy

The Need for PL

Everyone who has a dog is happy

Do

\therefore Obama is happy

The Need for PL

Everyone who has a dog is happy

Do

\therefore *Ho*

The Need for PL

$$\forall x(Dx \rightarrow Hx)$$

$$Do$$

$$\therefore Ho$$

Translation into PL

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Symbolization Keys

Predicate Logic: Symbolization Keys

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A : Abelard loves Heloise

H : Heloise loves Abelard

B : Abelard is bald

Predicate Logic: Symbolization Keys

- We translated into SL with a *symbolization key*, which told us, for every relevant statement letter, which statement of English it represented.
- ▷ For instance:
 - A : Abelard loves Heloise
 - H : Heloise loves Abelard
 - B : Abelard is bald
- ▷ We will also translate into PL with a *symbolization key*, except that these symbolization keys will tell us what each relevant *name* and *predicate* of PL means.

Predicate Logic: Names

- In PL, we use the lowercase letters ‘ a ’ through ‘ v ’ as *names*.
(We can add subscripts if we need to.)

$a, b, c, d, \dots, t, u, v, a_1, b_1, c_1, \dots$

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$a, b, c, d, \dots, t, u, v, a_1, b_1, c_1, \dots$

- The names in PL are just like *proper names* in English. Each name in PL refers to some particular person, place or thing.

Predicate Logic: Names

- A (partial) symbolization key:

a : Abelard

h : Heloise

b : Barcelona

j : Jupiter

Predicate Logic: Predicates

- In PL, we use *uppercase* letters, 'A' through 'Z', for *predicates*. (We can add subscripts if we need to.)

A, B, C, D, . . . , X, Y, Z, A₁, B₁, C₁, . . .

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- Think of a *predicate* as a *gappy statement*—it's a statement with a name (or names) missing.

Tammy met Sammy at the mall.

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- Think of a *predicate* as a *gappy statement*—it's a statement with a name (or names) missing.

_____ met _____ at the mall.

Predicate Logic: Predicates

- A (partial) symbolization key:

L _____ : _____ is large

B _____ : _____ is bald

P _____ : _____ loves Philosophy

X _____ : _____ is excited

Predicate Logic: Names and Predicates

- Predicates are statements with gaps.

Predicate Logic: Names and Predicates

- ▶ Predicates are statements with gaps.
- ▶ If will fill in those gaps with names, then we get back a statement.

Predicate Logic: Names and Predicates

a : Abelard	L _____	: _____ is large
h : Heloise	B _____	: _____ is bald
b : Barcelona	P _____	: _____ loves Philosophy
j : Jupiter	X _____	: _____ is excited

► Abelard is bald :

Predicate Logic: Names and Predicates

a : Abelard L _____ : _____ is large
 h : Heloise B _____ : _____ is bald
 b : Barcelona P _____ : _____ loves Philosophy
 j : Jupiter X _____ : _____ is excited

► Abelard is bald : Ba

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- ▶ Abelard is bald : Ba
- ▶ Heloise is excited :

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- ▶ Abelard is bald : Ba
- ▶ Heloise is excited : Xh
- ▶ Heloise isn't bald :

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- ▶ Abelard is bald : Ba
- ▶ Heloise is excited : Xh
- ▶ Heloise isn't bald : $\neg Bh$
- ▶ Abelard and Heloise love Philosophy :

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- ▶ Heloise is excited : Xh
- ▶ Heloise isn't bald : $\neg Bh$
- ▶ Abelard and Heloise love Philosophy : $Pa \wedge Ph$

Predicate Logic: Names and Predicates

a : Abelard L _____ : _____ is large
 h : Heloise B _____ : _____ is bald
 b : Barcelona P _____ : _____ loves Philosophy
 j : Jupiter X _____ : _____ is excited

- ▶ Heloise is excited only if Abelard loves Philosophy :

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$Xh \rightarrow Pa$

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b : Barcelona P _____ : _____ loves Philosophy

j : Jupiter X _____ : _____ is excited

- ▶ Heloise is excited only if Abelard loves Philosophy :

$Xh \rightarrow Pa$

- ▶ Barcelona is large unless Jupiter isn't. :

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 h : Heloise B _____ : _____ is bald
 b : Barcelona P _____ : _____ loves Philosophy
 j : Jupiter X _____ : _____ is excited

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- ▶ Barcelona is large unless Jupiter isn't. : $Lb \vee \neg Lj$

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 $\neg Pa \rightarrow \neg Xh$

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 $Xh \rightarrow Pa$
- ▶ Barcelona is large unless Jupiter isn't. : $Lb \vee \neg Lj$
- ▶ Heloise isn't excited if Abelard doesn't love Philosophy :
 $\neg Pa \rightarrow \neg Xh$
- ▶ Neither Barcelona nor Jupiter is large :

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- ▶ Heloise is excited only if Abelard loves Philosophy :
 $Xh \rightarrow Pa$
- ▶ Barcelona is large unless Jupiter isn't. : $Lb \vee \neg Lj$
- ▶ Heloise isn't excited if Abelard doesn't love Philosophy :
 $\neg Pa \rightarrow \neg Xh$
- ▶ Neither Barcelona nor Jupiter is large : $\neg(Lb \vee Lj)$

- Recall from SL:

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 - ▷ A unless B : $A \vee B$

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 - ▷ \mathcal{A} unless \mathcal{B} : $\mathcal{A} \vee \mathcal{B}$
 - ▷ \mathcal{A} only if \mathcal{B} : $\mathcal{A} \rightarrow \mathcal{B}$

- Recall from SL:
 - ▷ \mathcal{A} unless \mathcal{B} : $\mathcal{A} \vee \mathcal{B}$
 - ▷ \mathcal{A} only if \mathcal{B} : $\mathcal{A} \rightarrow \mathcal{B}$
 - ▷ Neither \mathcal{A} nor \mathcal{B} : $\neg(\mathcal{A} \vee \mathcal{B})$

Predicate Logic: Variables

- Predicates are statements with gaps for names. Putting a name in the gap gives us a statement. However, we will also allow ourselves to fill the gap in a predicate with a *variable*.

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- In PL, the lowercase letters $w, x, y,$ and z are *variables*. (We can add subscripts if we need to.)

$w, x, y, z, w_1, x_1, y_1, z_1, w_2, \dots$

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- In PL, the lowercase letters w , x , y , and z are *variables*. (We can add subscripts if we need to.)

$w, x, y, z, w_1, x_1, y_1, z_1, w_2, \dots$

- ▶ Think of a variable as a name without a fixed meaning—it can refer to *anything* (in the domain).

Predicate Logic: Variables and quantifiers

- Variables in PL are a bit like 'one' in formal English.
One should be circumspect when meeting in-laws.

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Predicate Logic: Variables and Quantifiers

S _____ : _____ should be circumspect when meeting in-laws.

Predicate Logic: Variables and Quantifiers

S _____ : _____ should be circumspect when meeting in-laws.

- ▶ One should be circumspect when meeting in-laws

Sx

Predicate Logic: Variables and Quantifiers

S _____ : _____ should be circumspect when meeting in-laws.

- ▶ One should be circumspect when meeting in-laws

Sx

- ▶ Everyone should be circumspect when meeting in-laws

$\forall x Sx$

Predicate Logic: Variables and Quantifiers

S _____ : _____ should be circumspect when meeting in-laws.

- ▶ One should be circumspect when meeting in-laws

Sx

- ▶ Everyone should be circumspect when meeting in-laws

$\forall x Sx$

- ▶ Someone should be circumspect when meeting in-laws

$\exists x Sx$

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 - Any x makes ' A_x ' true.

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- And ' $\exists x A_x$ ' says that ' A_x ' is true when we let ' x ' refer to *some* thing.

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 - Some x makes ' A_x ' true.

Predicacte Logic: Domains

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- A *Domain* specifies which things we might be talking about. It says which things a variable in our language could refer to.

Predicacte Logic: Domains

- Variables and Quantifiers require us to add one further thing to our symbolization keys.
- We must say which things our variables could refer to—which things we are potentially talking about.
- A *Domain* specifies which things we might be talking about. It says which things a variable in our language could refer to.
- ▶ Note: if one of our *names* refers to something, then that thing must be included in the domain.

Predicate Logic: Domains

Domain : all students at Pitt

H _____ : _____ is happy

D _____ : _____ has a dog

Predicate Logic: Domains

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- ▶ Every student at Pitt has a dog.

$\forall x D x$

Predicate Logic: Domains

Domain : all students at Pitt

H _____ : _____ is happy

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- ▶ Every student at Pitt has a dog.

$\forall x Dx$

- ▶ Obama is happy.

Predicate Logic: Domains

Domain : all students at Pitt

H _____ : _____ is happy

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o : Obama

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$\forall x Dx$

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Predicate Logic: Domains

Domain : all people

H _____ : _____ is happy

D _____ : _____ has a dog

o : Obama

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$\forall x Dx$

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Predicate Logic: Domains

Domain : all people

D _____ : _____ has a dog

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$$\forall x (Px \rightarrow Dx)$$

Predicate Logic: Domains

Domain : all people

D _____ : _____ has a dog

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- ▶ Every student at Pitt has a dog.

$$\forall x (Px \rightarrow Dx)$$

- ▶ Some student at Pitt has a dog.

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P _____ : _____ is a student at Pitt

- ▶ Every student at Pitt has a dog.

$$\forall x (Px \rightarrow Dx)$$

- ▶ Some student at Pitt has a dog.

$$\exists x (Px \wedge Dx)$$

Predicate Logic: Domains

Domain : all people

*B*_____ : _____ is bald

Predicate Logic: Domains

Domain : all people

B _____ : _____ is bald

- ▷ Everyone is bald.

Predicate Logic: Domains

Domain : all people

B _____ : _____ is bald

- ▷ Everyone is bald.

$\forall x Bx$

Predicate Logic: Domains

Domain : all things on planet Earth

B _____ : _____ is bald

- ▷ Everyone is bald.

$\forall x Bx$

Predicate Logic: Domains

Domain : all things on planet Earth

B _____ : _____ is bald

P _____ : _____ is a person

- ▶ Everyone is bald.

$\forall x Bx$

Predicate Logic: Domains

Domain : all things on planet Earth

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 - Each name has to refer to one and only one thing
 - Multiple names can refer to the same thing (e.g. 'Sam Clemens' and 'Mark Twain')
- For each relevant **predicate** of PL, it tells us which gappy statement that predicate represents.

Translation into PL

Important Statement Forms

Four Important Statement Forms

All F s are G s

Four Important Statement Forms

All F s are G s

No F s are G s

Four Important Statement Forms

All F s are G s

No F s are G s

Some F s are G s

Four Important Statement Forms

All F s are G s

No F s are G s

Some F s are G s

Some F s are not G s

Four Important Statement Forms

All F s are G s

- ▶ All mammals are warm-blooded

No F s are G s

Some F s are G s

Some F s are not G s

Four Important Statement Forms

All F s are G s

- ▶ All mammals are warm-blooded

No F s are G s

- ▶ No reptiles are warm-blooded

Some F s are G s

Some F s are not G s

Four Important Statement Forms

All F s are G s

- ▶ All mammals are warm-blooded

No F s are G s

- ▶ No reptiles are warm-blooded

Some F s are G s

- ▶ Some mammals are carnivorous

Some F s are not G s

Four Important Statement Forms

All F s are G s

- ▶ All mammals are warm-blooded

No F s are G s

- ▶ No reptiles are warm-blooded

Some F s are G s

- ▶ Some mammals are carnivorous

Some F s are not G s

- ▶ Some mammals are not carnivorous

All \mathcal{F} s are \mathcal{G} s

- All of the following mean the same thing, and so can be translated into PL in the same way:

All \mathcal{F} s are \mathcal{G} s

- All of the following mean the same thing, and so can be translated into PL in the same way:
 - ▷ All \mathcal{F} s are \mathcal{G} s All mammals are warm-blooded

All \mathcal{F} s are \mathcal{G} s

- All of the following mean the same thing, and so can be translated into PL in the same way:
 - ▷ All \mathcal{F} s are \mathcal{G} s All mammals are warm-blooded
 - ▷ Every \mathcal{F} is \mathcal{G} Every mammal is warm-blooded

All \mathcal{F} s are \mathcal{G} s

- All of the following mean the same thing, and so can be translated into PL in the same way:
 - ▷ All \mathcal{F} s are \mathcal{G} s All mammals are warm-blooded
 - ▷ Every \mathcal{F} is \mathcal{G} Every mammal is warm-blooded
 - ▷ Each \mathcal{F} is \mathcal{G} Each mammal is warm-blooded

All \mathcal{F} s are \mathcal{G} s

- All of the following mean the same thing, and so can be translated into PL in the same way:
 - ▷ All \mathcal{F} s are \mathcal{G} s All mammals are warm-blooded
 - ▷ Every \mathcal{F} is \mathcal{G} Every mammal is warm-blooded
 - ▷ Each \mathcal{F} is \mathcal{G} Each mammal is warm-blooded
 - ▷ Any \mathcal{F} is \mathcal{G} Any mammal is warm-blooded

All \mathcal{F} s are \mathcal{G} s

- Given a domain,

$$\forall x(\mathcal{F}x \rightarrow \mathcal{G}x)$$

says:

All \mathcal{F} s in the domain are \mathcal{G}

All \mathcal{F} s are \mathcal{G} s

- Given a domain,

$$\forall x(\mathcal{F}x \rightarrow \mathcal{G}x)$$

says:

All \mathcal{F} s in the domain are \mathcal{G}

Every \mathcal{F} in the domain is \mathcal{G}

All \mathcal{F} s are \mathcal{G} s

- Given a domain,

$$\forall x(\mathcal{F}x \rightarrow \mathcal{G}x)$$

says:

All \mathcal{F} s in the domain are \mathcal{G}

Every \mathcal{F} in the domain is \mathcal{G}

Each \mathcal{F} in the domain is \mathcal{G}

All \mathcal{F} s are \mathcal{G} s

- Given a domain,

$$\forall x(\mathcal{F}x \rightarrow \mathcal{G}x)$$

says:

All \mathcal{F} s in the domain are \mathcal{G}

Every \mathcal{F} in the domain is \mathcal{G}

Each \mathcal{F} in the domain is \mathcal{G}

Any \mathcal{F} in the domain is \mathcal{G}

All \mathcal{F} s are \mathcal{G} s

- Given a domain,

$$\forall y(\mathcal{F}y \rightarrow \mathcal{G}y)$$

says:

All \mathcal{F} s in the domain are \mathcal{G}

Every \mathcal{F} in the domain is \mathcal{G}

Each \mathcal{F} in the domain is \mathcal{G}

Any \mathcal{F} in the domain is \mathcal{G}

All \mathcal{F} s are \mathcal{G} s

- Given a domain,

$$\forall z(\mathcal{F}z \rightarrow \mathcal{G}z)$$

says:

All \mathcal{F} s in the domain are \mathcal{G}

Every \mathcal{F} in the domain is \mathcal{G}

Each \mathcal{F} in the domain is \mathcal{G}

Any \mathcal{F} in the domain is \mathcal{G}

All mammals are warm-blooded

Domain : all mammals

W _____ : _____ is warm-blooded

$\forall y Wy$

All mammals are warm-blooded

Domain : all animals

W _____ : _____ is warm-blooded

$\forall y Wy$

All mammals are warm-blooded

Domain : all animals

W _____ : _____ is warm-blooded

M _____ : _____ is a mammal

$\forall y Wy$

All mammals are warm-blooded

Domain : all animals

W _____ : _____ is warm-blooded

M _____ : _____ is a mammal

$$\forall x(Mx \rightarrow Wx)$$

No F s are G s

- All of the following mean the same thing, and so can be translated into PL in the same way:

No F s are G s

- All of the following mean the same thing, and so can be translated into PL in the same way:
 - ▷ No F s are G s
 - No reptiles are warm-blooded

No F s are G s

- All of the following mean the same thing, and so can be translated into PL in the same way:
 - ▷ No F s are G s No reptiles are warm-blooded
 - ▷ No F is G No reptile is warm-blooded

No F s are G s

- All of the following mean the same thing, and so can be translated into PL in the same way:
 - ▷ No F s are G s No reptiles are warm-blooded
 - ▷ No F is G No reptile is warm-blooded
 - ▷ Every F is not G s Every reptile is not warm-blooded

No \mathcal{F} s are \mathcal{G} s

- Given a domain,

$$\forall x(\mathcal{F}x \rightarrow \neg\mathcal{G}x)$$

says:

No \mathcal{F} s in the domain are \mathcal{G}

No \mathcal{F} s are \mathcal{G} s

- Given a domain,

$$\forall x(\mathcal{F}x \rightarrow \neg\mathcal{G}x)$$

says:

No \mathcal{F} s in the domain are \mathcal{G}

No \mathcal{F} in the domain is \mathcal{G}

No \mathcal{F} s are \mathcal{G} s

- Given a domain,

$$\forall x(\mathcal{F}x \rightarrow \neg\mathcal{G}x)$$

says:

No \mathcal{F} s in the domain are \mathcal{G}

No \mathcal{F} in the domain is \mathcal{G}

Every \mathcal{F} in the domain is not \mathcal{G}

No \mathcal{F} s are \mathcal{G} s

- Given a domain,

$$\neg \exists x (\mathcal{F}x \wedge \mathcal{G}x)$$

says:

No \mathcal{F} s in the domain are \mathcal{G}

No \mathcal{F} in the domain is \mathcal{G}

Every \mathcal{F} in the domain is not \mathcal{G}

No reptiles are warm-blooded

Domain : all reptiles

W _____ : _____ is warm-blooded

$$\forall y \neg Wy$$

No reptiles are warm-blooded

Domain : all animals

W _____ : _____ is warm-blooded

$$\forall y \neg Wy$$

No reptiles are warm-blooded

Domain : all animals

W _____ : _____ is warm-blooded

R _____ : _____ is a reptile

$$\forall y \neg Wy$$

No reptiles are warm-blooded

Domain : all animals

W _____ : _____ is warm-blooded

R _____ : _____ is a reptile

$$\forall y (Ry \rightarrow \neg Wy)$$

Some \mathcal{F} s are \mathcal{G} s

- All of the following mean the same thing, and so can be translated into PL in the same way:

Some \mathcal{F} s are \mathcal{G} s

- All of the following mean the same thing, and so can be translated into PL in the same way:
 - ▶ Some \mathcal{F} s are \mathcal{G} s Some mammals are carnivorous

Some \mathcal{F} s are \mathcal{G} s

- All of the following mean the same thing, and so can be translated into PL in the same way:
 - ▷ Some \mathcal{F} s are \mathcal{G} s Some mammals are carnivorous
 - ▷ Some \mathcal{F} is \mathcal{G} Some mammal is carnivorous

Some \mathcal{F} s are \mathcal{G} s

- All of the following mean the same thing, and so can be translated into PL in the same way:
 - ▶ Some \mathcal{F} s are \mathcal{G} s Some mammals are carnivorous
 - ▶ Some \mathcal{F} is \mathcal{G} Some mammal is carnivorous
 - ▶ There are \mathcal{G} \mathcal{F} s There are carnivorous mammals

Some \mathcal{F} s are \mathcal{G} s

- Given a domain,

$$\exists x(\mathcal{F}x \wedge \mathcal{G}x)$$

says:

Some \mathcal{F} s in the domain are \mathcal{G}

Some \mathcal{F} s are \mathcal{G} s

- Given a domain,

$$\exists x(\mathcal{F}x \wedge \mathcal{G}x)$$

says:

Some \mathcal{F} s in the domain are \mathcal{G}

Some \mathcal{F} in the domain is \mathcal{G}

Some \mathcal{F} s are \mathcal{G} s

- Given a domain,

$$\exists x(\mathcal{F}x \wedge \mathcal{G}x)$$

says:

Some \mathcal{F} s in the domain are \mathcal{G}

Some \mathcal{F} in the domain is \mathcal{G}

There are \mathcal{G} \mathcal{F} s in the domain

Some \mathcal{F} s are \mathcal{G} s

- Given a domain,

$$\exists y(\mathcal{F}y \wedge \mathcal{G}y)$$

says:

Some \mathcal{F} s in the domain are \mathcal{G}

Some \mathcal{F} in the domain is \mathcal{G}

There are \mathcal{G} \mathcal{F} s in the domain

Some \mathcal{F} s are \mathcal{G} s

- Given a domain,

$$\exists z(\mathcal{F}z \wedge \mathcal{G}z)$$

says:

Some \mathcal{F} s in the domain are \mathcal{G}

Some \mathcal{F} in the domain is \mathcal{G}

There are \mathcal{G} \mathcal{F} s in the domain

Some mammals are carnivorous

Domain : all mammals

C _____ : _____ is carnivorous

$\exists z Cz$

Some mammals are carnivorous

Domain : all animals

C _____ : _____ is carnivorous

$\exists z Cz$

Some mammals are carnivorous

Domain : all animals

C _____ : _____ is carnivorous

M _____ : _____ is a mammal

$\exists z Cz$

Some mammals are carnivorous

Domain : all animals

C _____ : _____ is carnivorous

M _____ : _____ is a mammal

$$\exists x(Mx \wedge Cx)$$

Some \mathcal{F} s are not \mathcal{G} s

- All of the following mean the same thing, and so can be translated into PL in the same way:

Some \mathcal{F} s are not \mathcal{G} s

- All of the following mean the same thing, and so can be translated into PL in the same way:
 - ▷ Some \mathcal{F} s are not \mathcal{G} s Some mammals are not carnivorous

Some \mathcal{F} s are not \mathcal{G} s

- All of the following mean the same thing, and so can be translated into PL in the same way:
 - ▷ Some \mathcal{F} s are not \mathcal{G} s Some mammals are not carnivorous
 - ▷ Some \mathcal{F} is not \mathcal{G} Some mammal is not carnivorous

Some \mathcal{F} s are not \mathcal{G} s

- All of the following mean the same thing, and so can be translated into PL in the same way:
 - ▷ Some \mathcal{F} s are not \mathcal{G} s Some mammals are not carnivorous
 - ▷ Some \mathcal{F} is not \mathcal{G} Some mammal is not carnivorous
 - ▷ There are non- \mathcal{G} \mathcal{F} s There are non-carnivorous mammals

Some \mathcal{F} s are not \mathcal{G} s

- Given a domain,

$$\exists x(\mathcal{F}x \wedge \neg \mathcal{G}x)$$

says:

Some \mathcal{F} s in the domain are not \mathcal{G}

Some \mathcal{F} s are not \mathcal{G} s

- Given a domain,

$$\exists x(\mathcal{F}x \wedge \neg \mathcal{G}x)$$

says:

Some \mathcal{F} s in the domain are not \mathcal{G}

Some \mathcal{F} in the domain is not \mathcal{G}

Some \mathcal{F} s are not \mathcal{G} s

- Given a domain,

$$\exists x(\mathcal{F}x \wedge \neg \mathcal{G}x)$$

says:

Some \mathcal{F} s in the domain are not \mathcal{G}

Some \mathcal{F} in the domain is not \mathcal{G}

There are non- \mathcal{G} \mathcal{F} s in the domain

Some \mathcal{F} s are not \mathcal{G} s

- Given a domain,

$$\exists w(\mathcal{F}w \wedge \neg \mathcal{G}w)$$

says:

Some \mathcal{F} s in the domain are not \mathcal{G}

Some \mathcal{F} in the domain is not \mathcal{G}

There are non- \mathcal{G} \mathcal{F} s in the domain

Some \mathcal{F} s are not \mathcal{G} s

- Given a domain,

$$\exists z(\mathcal{F}z \wedge \neg \mathcal{G}z)$$

says:

Some \mathcal{F} s in the domain are not \mathcal{G}

Some \mathcal{F} in the domain is not \mathcal{G}

There are non- \mathcal{G} \mathcal{F} s in the domain

Some mammals are not carnivorous

Domain : all mammals

C _____ : _____ is carnivorous

$$\exists z \neg Cz$$

Some mammals are not carnivorous

Domain : all animals

C _____ : _____ is carnivorous

$$\exists z \neg Cz$$

Some mammals are not carnivorous

Domain : all animals

C _____ : _____ is carnivorous

M _____ : _____ is a mammal

$$\exists z \neg Cz$$

Some mammals are not carnivorous

Domain : all animals

C _____ : _____ is carnivorous

M _____ : _____ is a mammal

$$\exists x(Mx \wedge \neg Cx)$$

Four Important Statement Forms

▷ All \mathcal{F} s are \mathcal{G} s

$$\forall x(\mathcal{F}x \rightarrow \mathcal{G}x)$$

Four Important Statement Forms

▷ All \mathcal{F} s are \mathcal{G} s

$$\forall x(\mathcal{F}x \rightarrow \mathcal{G}x)$$

▷ No \mathcal{F} s are \mathcal{G} s

$$\forall x(\mathcal{F}x \rightarrow \neg \mathcal{G}x)$$

Four Important Statement Forms

▷ All \mathcal{F} s are \mathcal{G} s

$$\forall x(\mathcal{F}x \rightarrow \mathcal{G}x)$$

▷ No \mathcal{F} s are \mathcal{G} s

$$\forall x(\mathcal{F}x \rightarrow \neg \mathcal{G}x)$$

▷ Some \mathcal{F} s are \mathcal{G} s

$$\exists x(\mathcal{F}x \wedge \mathcal{G}x)$$

Four Important Statement Forms

▷ All \mathcal{F} s are \mathcal{G} s

$$\forall x(\mathcal{F}x \rightarrow \mathcal{G}x)$$

▷ No \mathcal{F} s are \mathcal{G} s

$$\forall x(\mathcal{F}x \rightarrow \neg\mathcal{G}x)$$

▷ Some \mathcal{F} s are \mathcal{G} s

$$\exists x(\mathcal{F}x \wedge \mathcal{G}x)$$

▷ Some \mathcal{F} s are not \mathcal{G} s

$$\exists x(\mathcal{F}x \wedge \neg\mathcal{G}x)$$