Predicate Logic

Natural Deduction

PHIL 500

Showing Neither a Tautology Nor a Contradiction

Every barber shaves all and only those who don't shave themselves

Showing Neither a Tautology Nor a Contradiction

Every barber shaves all and only those who don't shave themselves

$$\begin{aligned} \text{domain} &: \text{people} \\ Bx &: \_\_\__x \text{ is a barber} \\ Sxy &: \_\_\__x \text{ shaves } \_\_\__y \end{aligned}$$

Showing Neither a Tautology Nor a Contradiction

Every barber shaves all and only those who don't shave themselves

$$\begin{aligned} \text{domain} &: \text{people} \\ Bx &: \underline{\quad}_x \text{ is a barber} \\ Sxy &: \underline{\quad}_x \text{ shaves } \underline{\quad}_y \end{aligned}$$

$$\forall x\,[Bx \rightarrow \forall y(Sxy \leftrightarrow \neg Syy)]$$

Showing Neither a Tautology Nor a Contradiction

Every barber shaves all and only those who don't shave themselves

$$\begin{aligned} \text{domain} &: \text{Billy} \\ B &: \text{Billy} \\ S &: \langle \text{ Billy, Billy } \rangle \end{aligned}$$

$$\forall x\,[Bx \rightarrow \forall y(Sxy \leftrightarrow \neg Syy)]$$

Showing Neither a Tautology Nor a Contradiction

Every barber shaves all and only those who don't shave themselves

$$\begin{aligned} \text{domain} &: \text{Billy} \\ B &: \text{Billy} \\ S &: \langle \text{ Billy, Billy } \rangle \end{aligned}$$

$$\forall x\,[Bx \longrightarrow \forall y(Sxy \longleftrightarrow \neg Syy)] \qquad [\ \text{F}\quad]$$

Showing Neither a Tautology Nor a Contradiction

Every barber shaves all and only those who don't shave themselves

$$\begin{aligned} \text{domain} &: \text{Billy} \\ B &: \\ S &: \end{aligned}$$

$$\forall x\, [Bx \rightarrow \forall y (Sxy \leftrightarrow \neg Syy)] \qquad [\ \textcolor{red}{\text{F}} \qquad]$$

Showing Neither a Tautology Nor a Contradiction

Every barber shaves all and only those who don't shave themselves

$$\begin{aligned} \text{domain} &: \text{Billy} \\ B &: \\ S &: \end{aligned}$$

$$\forall x\,[Bx \rightarrow \forall y(Sxy \leftrightarrow \neg Syy)] \qquad [\ \text{F},\ \text{T}\ ]$$

Contradiction

Billy shaves all and only those who don’t shave themselves

Contradiction

Billy shaves all and only those who don't shave themselves

$$\begin{aligned} \text{domain} &: \text{people} \\ Sxy &: \_\_\__x \text{ shaves } \_\_\__y \\ b &: \text{Billy} \end{aligned}$$

Contradiction

Billy shaves all and only those who don't shave themselves

$$\begin{aligned} \text{domain} &: \text{people} \\ Sxy &: \_\_\__x \text{ shaves } \_\_\__y \\ b &: \text{Billy} \end{aligned}$$

$$\forall y(Sby \leftrightarrow \neg Syy)$$

Contradiction

Billy shaves all and only those who don't shave themselves

$$\begin{aligned} \text{domain} &: 1 \\ S &: \\ b &: 1 \end{aligned}$$

$$\forall y(Sby \leftrightarrow \neg Syy)$$

Contradiction

Billy shaves all and only those who don't shave themselves

$$\begin{aligned} \text{domain} &: 1 \\ S &: \\ b &: 1 \end{aligned}$$

$$\forall y(Sby \leftrightarrow \neg Syy)$$

Contradiction

Billy shaves all and only those who don't shave themselves

$$\begin{aligned} \text{domain} &: 1 \\ S &: \\ b &: 1 \end{aligned}$$

$$\forall y(Sby \leftrightarrow \neg Syy)$$

Contradiction

Billy shaves all and only those who don't shave themselves

$$\begin{aligned} \text{domain} &: 1 \\ S &: \\ b &: 1 \\ y &: 1 \end{aligned}$$

$$\forall y(Sby \longleftrightarrow \neg Syy)$$

Contradiction

Billy shaves all and only those who don't shave themselves

$$\begin{aligned} \text{domain} &: 1 \\ S &: \langle 1, 1 \rangle ? \\ b &: 1 \\ y &: 1 \end{aligned}$$

$$\forall y(Sby \longleftrightarrow \neg Syy)$$

Natural Deduction for PL

Natural Deduction for PL

- To show that...

Natural Deduction for PL

- To show that...
  - ...an argument is an entailment

Natural Deduction for PL

- To show that...
  - ...an argument is an entailment
  - ...a collection of sentences is unsatisfiable

Natural Deduction for PL

- To show that...
  - ...an argument is an entailment
  - ...a collection of sentences is unsatisfiable
  - ...a sentence is a tautology

Natural Deduction for PL

- To show that...
  - ▹ ...an argument is an entailment
  - ▹ ...a collection of sentences is unsatisfiable
  - ▹ ...a sentence is a tautology
  - ▹ ...a sentence is a contradiction

Natural Deduction for PL

- To show that...
  - ...an argument is an entailment
  - ...a collection of sentences is unsatisfiable
  - ...a sentence is a tautology
  - ...a sentence is a contradiction
- we will provide a *natural deduction proof* (in PL)

Natural Deduction for PL

- The natural deduction system for PL includes *all the rules from SL*

Natural Deduction for PL

- The natural deduction system for PL includes *all the rules from SL*
  - This includes the derived rules

Natural Deduction for PL

- The natural deduction system for PL includes *all the rules from SL*
  - This includes the derived rules
- It includes *four new rules*

Natural Deduction for PL

- The natural deduction system for PL includes *all the rules from SL*
    - This includes the derived rules
- It includes *four new rules*
    - Universal Introduction ($\forall I$)

Natural Deduction for PL

- The natural deduction system for PL includes *all the rules from SL*
  - This includes the derived rules
- It includes *four new rules*
  - Universal Introduction ($\forall I$)
  - Universal Elimination ($\forall E$)

Natural Deduction for PL

- The natural deduction system for PL includes *all the rules from SL*
  - This includes the derived rules
- It includes *four new rules*
  - Universal Introduction ($\forall I$)
  - Universal Elimination ($\forall E$)
  - Existential Introduction ($\exists I$)

Natural Deduction for PL

- The natural deduction system for PL includes *all the rules from SL*
  - This includes the derived rules
- It includes *four new rules*
  - Universal Introduction ($\forall I$)
  - Universal Elimination ($\forall E$)
  - Existential Introduction ($\exists I$)
  - Existential Elimination ($\exists E$)

Outline

Universal Elimination

Existential Introduction

Universal Introduction

Existential Elimination

Universal Elimination

Universal Elimination

Universal Elimination ($\forall E$)

	$\forall x\, \mathscr{A}(\ldots x \ldots x \ldots)$
▹	$\mathscr{A}(\ldots n \ldots n \ldots)$

Universal Elimination

Universal Elimination ($\forall E$)

$$\forall x\, \mathscr{A}(\ldots x \ldots x \ldots)$$

$$\triangleright \quad \mathscr{A}(\ldots n \ldots n \ldots)$$

- '$\mathscr{A}(\ldots x \ldots x \ldots)$' stands for any sentence in which the variable 'x' may appear *free*.

Universal Elimination

Universal Elimination ($\forall E$)

$$\begin{array}{l|l} & \forall x\, \mathscr{A}(\ldots x \ldots x \ldots) \\ \triangleright & \mathscr{A}(\ldots n \ldots n \ldots) \end{array}$$

- '$\mathscr{A}(\ldots x \ldots x \ldots)$' stands for any sentence in which the variable 'x' may appear *free*.
- 'x' may appear free more than once.

Universal Elimination

Universal Elimination ($\forall E$)

$$\begin{array}{l|l} & \forall x\, \mathscr{A}(\ldots x \ldots x \ldots) \\ \triangleright & \mathscr{A}(\ldots n \ldots n \ldots) \end{array}$$

- '$\mathscr{A}(\ldots x \ldots x \ldots)$' stands for any sentence in which the variable 'x' may appear *free*.
- 'x' may appear free more than once.
- '$\mathscr{A}(\ldots n \ldots n \ldots)$' stands for the result of going through the sentence $\mathscr{A}$ and replacing ***every*** free occurrence of 'x' with the same *name* 'n'.

Universal Elimination

m. | $\forall x\,(Lhx \rightarrow Lxh)$

Universal Elimination

m.	$\forall x\,(Lhx \rightarrow Lxh)$	
k.	$Lha \rightarrow Lah$	$\forall E$ m

Universal Elimination

m.	$\forall x\,(Lhx \rightarrow Lxh)$	
k.	$Lha \rightarrow Lah$	$\forall E$ m

$$\mathscr{A}(\ldots x \ldots x \ldots) = Rhx \rightarrow Rxh$$
$$\mathscr{A}(\ldots n \ldots n \ldots) = Rha \rightarrow Rah$$

Universal Elimination

m. $\forall x\,(Lhx \rightarrow Lxh)$

Universal Elimination

m.	$\forall x\,(Lhx \rightarrow Lxh)$	
k.	$Lhh \rightarrow Lhh$	$\forall E$ m

Universal Elimination

m.	$\forall x\,(Lhx \to Lxh)$	
k.	$Lhh \to Lhh$	$\forall E$ m

$$\mathscr{A}(\ldots x \ldots x \ldots) = Rhx \to Rxh$$

$$\mathscr{A}(\ldots n \ldots n \ldots) = Rhh \to Rhh$$

$\forall y(Sby \leftrightarrow \neg Syy) \vdash \bot$

1	$\forall y(Sby \leftrightarrow \neg Syy)$

$\forall y(Sby \leftrightarrow \neg Syy) \vdash \bot$

1	$\forall y(Sby \leftrightarrow \neg Syy)$	
2	$Sbb \leftrightarrow \neg Sbb$	$\forall E$ 1

$\forall y(Sby \leftrightarrow \neg Syy) \vdash \bot$

1	$\forall y(Sby \leftrightarrow \neg Syy)$	
2	$Sbb \leftrightarrow \neg Sbb$	$\forall E$ 1
3	$\quad Sbb$	Ass ($\neg I$)

$\forall y(Sby \leftrightarrow \neg Syy) \vdash \bot$

1	$\forall y(Sby \leftrightarrow \neg Syy)$	
2	$Sbb \leftrightarrow \neg Sbb$	$\forall E$ 1
3	$\quad Sbb$	Ass ($\neg I$)
4	$\quad \neg Sbb$	$\leftrightarrow E$ 2, 3

$\forall y(Sby \leftrightarrow \neg Syy) \vdash \bot$

1	$\forall y(Sby \leftrightarrow \neg Syy)$	
2	$Sbb \leftrightarrow \neg Sbb$	$\forall E$ 1
3	$\quad Sbb$	Ass ($\neg I$)
4	$\quad \neg Sbb$	$\leftrightarrow E$ 2, 3
5	$\quad \bot$	$\bot I$ 3, 4

$\forall y(Sby \leftrightarrow \neg Syy) \vdash \bot$

1	$\forall y(Sby \leftrightarrow \neg Syy)$	
2	$Sbb \leftrightarrow \neg Sbb$	$\forall E$ 1
3	$\quad Sbb$	Ass ($\neg I$)
4	$\quad \neg Sbb$	$\leftrightarrow E$ 2, 3
5	$\quad \bot$	$\bot I$ 3, 4
6	$\neg Sbb$	$\neg I$ 3–5

$\forall y(Sby \leftrightarrow \neg Syy) \vdash \bot$

1	$\forall y(Sby \leftrightarrow \neg Syy)$	
2	$Sbb \leftrightarrow \neg Sbb$	$\forall E$ 1
3	Sbb	Ass ($\neg I$)
4	$\neg Sbb$	$\leftrightarrow E$ 2, 3
5	$\bot$	$\bot I$ 3, 4
6	$\neg Sbb$	$\neg I$ 3–5
7	Sbb	$\leftrightarrow E$ 2, 6

$\forall y(Sby \leftrightarrow \neg Syy) \vdash \bot$

1	$\forall y(Sby \leftrightarrow \neg Syy)$	
2	$Sbb \leftrightarrow \neg Sbb$	$\forall E$ 1
3	$\quad Sbb$	Ass ($\neg I$)
4	$\quad \neg Sbb$	$\leftrightarrow E$ 2, 3
5	$\quad \bot$	$\bot I$ 3, 4
6	$\neg Sbb$	$\neg I$ 3–5
7	Sbb	$\leftrightarrow E$ 2, 6
8	$\bot$	$\bot I$ 6, 7

$\forall y(Sby \leftrightarrow \neg Syy) \vdash \bot$

1	$\forall y(Sby \leftrightarrow \neg Syy)$	
2	$Sbb \leftrightarrow \neg Sbb$	$\forall E$ 1
3	$\quad Sbb$	Ass ($\neg I$)
4	$\quad \neg Sbb$	$\leftrightarrow E$ 2, 3
5	$\quad \bot$	$\bot I$ 3, 4
6	$\neg Sbb$	$\neg I$ 3–5
7	Sbb	$\leftrightarrow E$ 2, 6
8	$\bot$	$\bot I$ 6, 7

Proving a Sentence is a Contradiction

- So: $\forall y(Sby \leftrightarrow \neg Syy) \vdash \bot$

Proving a Sentence is a Contradiction

- So: $\forall y(Sby \leftrightarrow \neg Syy) \;\vdash\; \bot$
- So: $\forall y(Sby \leftrightarrow \neg Syy) \;\vDash\; \bot$

Proving a Sentence is a Contradiction

- So: $\forall y(Sby \leftrightarrow \neg Syy) \vdash \bot$
- So: $\forall y(Sby \leftrightarrow \neg Syy) \vDash \bot$
- So: there's no interpretation which makes $\forall y(Sby \leftrightarrow \neg Syy)$ true

Proving a Sentence is a Contradiction

- So: $\forall y(Sby \leftrightarrow \neg Syy) \;\vdash\; \bot$
- So: $\forall y(Sby \leftrightarrow \neg Syy) \;\models\; \bot$
- So: there's no interpretation which makes $\forall y(Sby \leftrightarrow \neg Syy)$ true
- So: $\forall y(Sby \leftrightarrow \neg Syy)$ is a contradiction.

Universal Elimination

1	$\forall z(Fz \to Gz)$

Universal Elimination

1	$\forall z(Fz \rightarrow Gz)$	
2	$Fa \rightarrow Gz$	$\forall E$ 1

Universal Elimination

1	$\forall z(Fz \rightarrow Gz)$	
2	$Fa \rightarrow Gz$	$\forall E$ 1 $\longleftarrow$ **MISTAKE!**

Universal Elimination

1	$\forall z(Fz \rightarrow Gz)$	
2	$Fa \rightarrow Gz$	$\forall E$ 1 $\longleftarrow$ **MISTAKE!**

- You must replace ***every*** free occurrence of 'z' with the same name.

Universal Elimination

1 | $\forall z(Fz \to Gz)$

Universal Elimination

1	$\forall z(Fz \rightarrow Gz)$	
2	$Fa \rightarrow Gb$	$\forall E$ 1

Universal Elimination

1	$\forall z(Fz \rightarrow Gz)$	
2	$Fa \rightarrow Gb$	$\forall E$ 1 $\longleftarrow$ **MISTAKE!**

Universal Elimination

1	$\forall z(Fz \rightarrow Gz)$		
2	$Fa \rightarrow Gb$	$\forall E$ 1	$\longleftarrow$ **MISTAKE!**

- You must replace every free occurrence of 'z' with the ***same*** name.

Universal Elimination

1	$\forall z(Fz \rightarrow \exists zGz)$

Universal Elimination

1	$\forall z(Fz \rightarrow \exists zGz)$	
2	$Fa \rightarrow \exists zGa$	$\forall E$ 1

Universal Elimination

1	$\forall z(Fz \rightarrow \exists zGz)$	
2	$Fa \rightarrow \exists zGa$	$\forall E$ 1 $\longleftarrow$ **MISTAKE!**

Universal Elimination

1	$\forall z(Fz \rightarrow \exists zGz)$		
2	$Fa \rightarrow \exists zGa$	$\forall E$ 1	$\longleftarrow$ **MISTAKE!**

- You must replace every ***free*** occurrence of 'z' with the same name.

Universal Elimination

1	$\forall x Fx \rightarrow Ga$

Universal Elimination

1	$\forall x Fx \rightarrow Ga$	
2	$Fa \rightarrow Ga$	$\forall E$ 1

Universal Elimination

1	$\forall x Fx \rightarrow Ga$		
2	$Fa \rightarrow Ga$	$\forall E$ 1	$\longleftarrow$ **MISTAKE!**

Universal Elimination

1	$\forall x Fx \rightarrow Ga$		
2	$Fa \rightarrow Ga$	$\forall E$ 1	$\longleftarrow$ **MISTAKE!**

- Rules may not be applied to sub-sentences.

Universal Elimination

1	$\forall x Fx \rightarrow Ga$	
2	$Fa \rightarrow Ga$	$\forall E$ 1 $\longleftarrow$ **MISTAKE!**

- *Rules may not be applied to sub-sentences.*

Rules may not be applied to sub-sentences

1	$(Pc \wedge Qc) \rightarrow Rj$

Rules may not be applied to sub-sentences

1	$(Pc \wedge Qc) \rightarrow Rj$	
2	Pc	$\wedge E$ 1

Rules may not be applied to sub-sentences

1	$(Pc \land Qc) \to Rj$	
2	Pc	$\land E$ 1 $\longleftarrow$ **MISTAKE!**

Rules may not be applied to sub-sentences

1	$(Fa \rightarrow Gc) \rightarrow Ha$
2	Fa

Rules may not be applied to sub-sentences

1	$(Fa \rightarrow Gc) \rightarrow Ha$	
2	Fa	
3	$Gc \rightarrow Ha$	$\rightarrow E$ 1, 2

Rules may not be applied to sub-sentences

1	$(Fa \rightarrow Gc) \rightarrow Ha$	
2	Fa	
3	$Gc \rightarrow Ha$	$\rightarrow E$ 1, 2 $\longleftarrow$ **MISTAKE!**

$\vdash \forall x \forall y\, Rxy \rightarrow Rjj$

1

$\vdash \forall x \forall y\, Rxy \rightarrow Rjj$

1	$\forall x \forall y\, Rxy$	Ass ($\rightarrow I$)

$\vdash \forall x \forall y\, Rxy \to Rjj$

1	$\forall x \forall y\, Rxy$	Ass ($\to I$)
2	$\forall y\, Rjy$	$\forall E$ 1

$\vdash \forall x \forall y\, Rxy \rightarrow Rjj$

1	$\forall x \forall y\, Rxy$	Ass ($\rightarrow I$)
2	$\forall y\, Rjy$	$\forall E$ 1
3	Rjj	$\forall E$ 2

$\vdash \forall x \forall y\, Rxy \rightarrow Rjj$

1	$\forall x \forall y\, Rxy$	Ass ($\rightarrow I$)
2	$\forall y\, Rjy$	$\forall E$ 1
3	Rjj	$\forall E$ 2
4	$\forall x \forall y\, Rxy \rightarrow Rjj$	$\rightarrow I$ 1–3

$\vdash \forall x \forall y\, Rxy \rightarrow Rjj$

1	$\forall x \forall y\, Rxy$	Ass ($\rightarrow I$)
2	$\forall y\, Rjy$	$\forall E$ 1
3	Rjj	$\forall E$ 2
4	$\forall x \forall y\, Rxy \rightarrow Rjj$	$\rightarrow I$ 1–3

Proving a Sentence is a Tautology

- So: $\vdash \forall x \forall y\, Rxy \rightarrow Rjj$

Proving a Sentence is a Tautology

- So: $\vdash \forall x \forall y\, Rxy \rightarrow Rjj$
- So: $\models \forall x \forall y\, Rxy \rightarrow Rjj$

Proving a Sentence is a Tautology

- So: $\vdash \forall x \forall y\, Rxy \rightarrow Rjj$
- So: $\models \forall x \forall y\, Rxy \rightarrow Rjj$
- So: there's no interpretation which makes $\forall x \forall y\, Rxy \rightarrow Rjj$ false.

Proving a Sentence is a Tautology

- So: $\vdash \forall x \forall y\, Rxy \rightarrow Rjj$
- So: $\models \forall x \forall y\, Rxy \rightarrow Rjj$
- So: there's no interpretation which makes $\forall x \forall y\, Rxy \rightarrow Rjj$ false.
- So: $\forall x \forall y\, Rxy \rightarrow Rjj$ is a tautology.

Existential Introduction

Existential Introduction

Existential Introduction ($\exists I$)

$$\mathscr{A}(\ldots n \ldots n \ldots)$$

$$\triangleright \quad \exists x\, \mathscr{A}(\ldots n \ldots x \ldots)$$

Existential Introduction

Existential Introduction ($\exists I$)

$$\mathscr{A}(\ldots n \ldots n \ldots)$$

$$\triangleright \quad \exists x\, \mathscr{A}(\ldots n \ldots x \ldots)$$

- '$\mathscr{A}(\ldots n \ldots n \ldots)$' stands for any sentence in which the name n appears.

Existential Introduction

Existential Introduction ($\exists I$)

$$\mathscr{A}(\ldots n \ldots n \ldots)$$

$$\triangleright \quad \exists x\, \mathscr{A}(\ldots n \ldots x \ldots)$$

- '$\mathscr{A}(\ldots n \ldots n \ldots)$' stands for any sentence in which the name n appears.
- n may appear more than once.

Existential Introduction

Existential Introduction ($\exists I$)

$$\begin{array}{l|l} & \mathscr{A}(\ldots n \ldots n \ldots) \\ \triangleright & \exists x\, \mathscr{A}(\ldots n \ldots x \ldots) \end{array}$$

- '$\mathscr{A}(\ldots n \ldots n \ldots)$' stands for any sentence in which the name n appears.
- n may appear more than once.
- '$\mathscr{A}(\ldots n \ldots x \ldots)$' stands for the result of going through the sentence $\mathscr{A}$ and replacing as many occurrences of 'n' with the *variable* x as you like (you don't have to replace *all* of them with the variable x).

Existential Introduction

m. | $(Fa \wedge Raa) \vee Gaa$

Existential Introduction

m.	$(Fa \wedge Raa) \vee Gaa$	
k.	$\exists z[(Fz \wedge Raz) \vee Gaa]$	$\exists I$ m

Existential Introduction

m.	$(Fa \wedge Raa) \vee Gaa$	
k.	$\exists z[(Fz \wedge Raz) \vee Gaa]$	$\exists I$ m

$$\mathscr{A}(\ldots n \ldots n \ldots) = (Fa \wedge Raa) \vee Gaa$$

$$\mathscr{A}(\ldots x \ldots n \ldots) = (Fz \wedge Raz) \vee Gaa$$

Existential Introduction

m.		$(Fa \wedge Raa) \vee Gaa$

Existential Introduction

m.	$(Fa \wedge Raa) \vee Gaa$	
k.	$\exists z[(Fa \wedge Rza) \vee Gza]$	$\exists I$ m

Existential Introduction

m.	$(Fa \wedge Raa) \vee Gaa$	
k.	$\exists z[(Fa \wedge Rza) \vee Gza]$	$\exists I$ m

$$\mathscr{A}(\ldots n \ldots n \ldots) = (Fa \wedge Raa) \vee Gaa$$

$$\mathscr{A}(\ldots x \ldots n \ldots) = (Fa \wedge Rza) \vee Gza$$

Existential Introduction

m.		$(Fa \wedge Raa) \vee Gaa$

Existential Introduction

m.	$(Fa \wedge Raa) \vee Gaa$	
k.	$\exists z[(Fz \wedge Rzz) \vee Gzz]$	$\exists I$ m

Existential Introduction

m.	$(Fa \wedge Raa) \vee Gaa$	
k.	$\exists z[(Fz \wedge Rzz) \vee Gzz]$	$\exists I$ m

$$\mathscr{A}(\ldots n \ldots n \ldots) = (Fa \wedge Raa) \vee Gaa$$

$$\mathscr{A}(\ldots x \ldots n \ldots) = (Fz \wedge Rzz) \vee Gzz$$

$\forall x\, Rxx \vdash \exists x\, \exists y\, Rxy$

1	$\forall x\, Rxx$

$\forall x\, Rxx \vdash \exists x\, \exists y\, Rxy$

1	$\forall x\, Rxx$	
2	Ree	$\forall E$ 1

$\forall x\, Rxx \vdash \exists x\, \exists y\, Rxy$

1	$\forall x\, Rxx$	
2	Ree	$\forall E$ 1
3	$\exists y\, Rey$	$\exists I$ 2

$\forall x\, Rxx \vdash \exists x\, \exists y\, Rxy$

1	$\forall x\, Rxx$	
2	Ree	$\forall E$ 1
3	$\exists y\, Rey$	$\exists I$ 2
4	$\exists x\, \exists y\, Rxy$	$\exists I$ 3

$\forall x\, Rxx \;\vdash\; \exists x\, \exists y\, Rxy$

1	$\forall x\, Rxx$	
2	Ree	$\forall E$ 1
3	$\exists y\, Rey$	$\exists I$ 2
4	$\exists x\, \exists y\, Rxy$	$\exists I$ 3

Existential Introduction

1 | Rab

Existential Introduction

1	Rab	
2	$\exists z\, Rzz$	$\exists I$ 1

Existential Introduction

1	Rab	
2	$\exists z\, Rzz$	$\exists I$ 1 $\longleftarrow$ **MISTAKE!**

Existential Introduction

1	Rab	
2	$\exists z\, Rzz$	$\exists I$ 1 $\longleftarrow$ **MISTAKE!**

- Each occurrence of the variable '*z*' must replace the ***same*** name.

Existential Introduction

1 | Rab

Existential Introduction

1	Rab	
2	$\exists z\, Raz$	$\exists I$ 1

Existential Introduction

1	Rab	
2	$\exists z\, Raz$	$\exists I$ 1
3	$\exists y\, \exists z\, Ryz$	$\exists I$ 2

$\forall x \forall y\, Zxy \;\vdash\; \exists w\, Zww$

$$
\begin{array}{lll}
1 & \vert & \forall x \forall y\, Zxy \\
\end{array}
$$

$\forall x \forall y Zxy \vdash \exists w Zww$

1	$\forall x \forall y Zxy$	
2	$\forall y Zny$	$\forall E$ 1

$\forall x \forall y Zxy \vdash \exists w Zww$

1	$\forall x \forall y Zxy$	
2	$\forall y Zny$	$\forall E$ 1
3	Znn	$\forall E$ 2

$\forall x \forall y Zxy \vdash \exists w Zww$

1	$\forall x \forall y Zxy$	
2	$\forall y Zny$	$\forall E$ 1
3	Znn	$\forall E$ 2
4	$\exists w Zww$	$\exists I$ 3

$\forall x\, \forall y\, Zxy \;\vdash\; \exists w\, Zww$

1	$\forall x\, \forall y\, Zxy$	
2	$\forall y\, Zny$	$\forall E$ 1
3	Znn	$\forall E$ 2
4	$\exists w\, Zww$	$\exists I$ 3

Universal Introduction

Universal Introduction

Universal Introduction ($\forall I$)

$$\mathscr{A}(\ldots n \ldots n \ldots)$$

$$\triangleright \quad \forall x\, \mathscr{A}(\ldots x \ldots x \ldots)$$

provided that: n *does not appear in any open assumption*

- An assumption is *open* at line *k* iff its vertical scope line extends to line *k*.

Universal Introduction

Universal Introduction ($\forall I$)

$$\begin{array}{l|l} & \mathscr{A}(\ldots n \ldots n \ldots) \\ \triangleright & \forall x\, \mathscr{A}(\ldots x \ldots x \ldots) \end{array}$$

provided that: n *does not appear in any open assumption*

- An assumption is *open* at line *k* iff its vertical scope line extends to line *k*.
- The name n cannot appear anywhere in $\mathscr{A}(\ldots x \ldots x \ldots)$.

Universal Introduction

1 | $\exists x\, Rxa$

Universal Introduction

1	$\exists x\, Rxa$	
2	$\forall y\, \exists x\, Rxy$	$\forall I$ 1

Universal Introduction

1	$\exists x\, Rxa$	
2	$\forall y\, \exists x\, Rxy$	$\forall I$ 1 $\longleftarrow$ **MISTAKE!**

Universal Introduction

1	$\exists x\, Rxa$		
2	$\forall y\, \exists x\, Rxy$	$\forall I$ 1	$\longleftarrow$ **MISTAKE!**

- The name '*a*' appears in the open assumption on line 1

Universal Introduction

1 | $\forall x\, Rxx$

Universal Introduction

1	$\forall x\, Rxx$	
2	Rcc	$\forall E$ 1

Universal Introduction

1	$\forall x\, Rxx$		
2	Rcc	$\forall E$ 1
3	$\forall x\, Rxc$	$\forall I$ 2

Universal Introduction

1	$\forall x\, Rxx$		
2	Rcc	$\forall E$ 1	
3	$\forall x\, Rxc$	$\forall I$ 2	$\longleftarrow$ **MISTAKE!**

Universal Introduction

1	$\forall x\, Rxx$	
2	Rcc	$\forall E$ 1
3	$\forall x\, Rxc$	$\forall I$ 2 $\longleftarrow$ **MISTAKE!**

- We need to replace ***every*** occurrence of '*c*' with '*x*'.

$\forall y\, Lay \vdash \forall x\, \exists y\, Lyx$

1	$\forall y\, Lay$

$\forall y\, Lay \vdash \forall x \exists y\, Lyx$

1	$\forall y\, Lay$	
2	Lab	$\forall E$ 1

$\forall y\, Lay \vdash \forall x\, \exists y\, Lyx$

1	$\forall y\, Lay$	
2	Lab	$\forall E$ 1
3	$\exists y\, Lyb$	$\exists I$ 2

$\forall y\, Lay \vdash \forall x\, \exists y\, Lyx$

1	$\forall y\, Lay$	
2	Lab	$\forall E$ 1
3	$\exists y\, Lyb$	$\exists I$ 2
4	$\forall x\, \exists y\, Lyx$	$\forall I$

$\forall y\, Lay \vdash \forall x\, \exists y\, Lyx$

1	$\forall y\, Lay$	
2	Lab	$\forall E$ 1
3	$\exists y\, Lyb$	$\exists I$ 2
4	$\forall x\, \exists y\, Lyx$	$\forall I$

A Syllogism

All cats are fluffy. Nothing fluffy is scary. So no cats are scary.

A Syllogism

All cats are fluffy. Nothing fluffy is scary. So no cats are scary.

$$\begin{aligned} \text{domain} &: \text{all things} \\ Cx &: \underline{\quad}_x \ \text{is a cat} \\ Fx &: \underline{\quad}_x \ \text{is fluffy} \\ Sx &: \underline{\quad}_x \ \text{is scary} \end{aligned}$$

A Syllogism

All cats are fluffy. Nothing fluffy is scary. So no cats are scary.

$$\begin{aligned} \text{domain} &: \text{all things} \\ Cx &: \_\_\__x \text{ is a cat} \\ Fx &: \_\_\__x \text{ is fluffy} \\ Sx &: \_\_\__x \text{ is scary} \end{aligned}$$

$$\forall x(Cx \rightarrow Fx), \forall y(Fy \rightarrow \neg Sy) \ \therefore \ \forall z(Cz \rightarrow \neg Sz)$$

$\forall x(Cx \rightarrow Fx), \forall y(Fy \rightarrow \neg Sy) \vdash \forall z(Cz \rightarrow \neg Sz)$

1	$\forall x(Cx \rightarrow Fx)$
2	$\forall y(Fy \rightarrow \neg Sy)$

$\forall x(Cx \rightarrow Fx), \forall y(Fy \rightarrow \neg Sy) \vdash \forall z(Cz \rightarrow \neg Sz)$

1	$\forall x(Cx \rightarrow Fx)$	
2	$\forall y(Fy \rightarrow \neg Sy)$	
3	Cb	Ass ($\rightarrow I$)

$\forall x(Cx \rightarrow Fx), \forall y(Fy \rightarrow \neg Sy) \vdash \forall z(Cz \rightarrow \neg Sz)$

1	$\forall x(Cx \rightarrow Fx)$	
2	$\forall y(Fy \rightarrow \neg Sy)$	
3	$\quad Cb$	Ass ($\rightarrow I$)
4	$\quad Cb \rightarrow Fb$	$\forall E$ 1

$\forall x(Cx \rightarrow Fx),\ \forall y(Fy \rightarrow \neg Sy) \vdash \forall z(Cz \rightarrow \neg Sz)$

1	$\forall x(Cx \rightarrow Fx)$	
2	$\forall y(Fy \rightarrow \neg Sy)$	
3	$\quad Cb$	Ass $(\rightarrow I)$
4	$\quad Cb \rightarrow Fb$	$\forall E$ 1
5	$\quad Fb$	$\rightarrow E$ 3, 4

$\forall x(Cx \rightarrow Fx), \forall y(Fy \rightarrow \neg Sy) \vdash \forall z(Cz \rightarrow \neg Sz)$

1	$\forall x(Cx \rightarrow Fx)$	
2	$\forall y(Fy \rightarrow \neg Sy)$	
3	Cb	Ass $(\rightarrow I)$
4	$Cb \rightarrow Fb$	$\forall E$ 1
5	Fb	$\rightarrow E$ 3, 4
6	$Fb \rightarrow \neg Sb$	$\rightarrow E$ 2

$\forall x(Cx \to Fx), \forall y(Fy \to \neg Sy) \vdash \forall z(Cz \to \neg Sz)$

1	$\forall x(Cx \to Fx)$	
2	$\forall y(Fy \to \neg Sy)$	
3	Cb	Ass $(\to I)$
4	$Cb \to Fb$	$\forall E$ 1
5	Fb	$\to E$ 3, 4
6	$Fb \to \neg Sb$	$\to E$ 2
7	$\neg Sb$	$\to E$ 5, 6

$\forall x(Cx \rightarrow Fx), \forall y(Fy \rightarrow \neg Sy) \vdash \forall z(Cz \rightarrow \neg Sz)$

1	$\forall x(Cx \rightarrow Fx)$	
2	$\forall y(Fy \rightarrow \neg Sy)$	
3	Cb	Ass ($\rightarrow I$)
4	$Cb \rightarrow Fb$	$\forall E$ 1
5	Fb	$\rightarrow E$ 3, 4
6	$Fb \rightarrow \neg Sb$	$\rightarrow E$ 2
7	$\neg Sb$	$\rightarrow E$ 5, 6
8	$Cb \rightarrow \neg Sb$	$\rightarrow I$ 3–7

$\forall x(Cx \rightarrow Fx),\ \forall y(Fy \rightarrow \neg Sy) \vdash \forall z(Cz \rightarrow \neg Sz)$

1	$\forall x(Cx \rightarrow Fx)$	
2	$\forall y(Fy \rightarrow \neg Sy)$	
3	$\quad Cb$	Ass ($\rightarrow I$)
4	$\quad Cb \rightarrow Fb$	$\forall E$ 1
5	$\quad Fb$	$\rightarrow E$ 3, 4
6	$\quad Fb \rightarrow \neg Sb$	$\rightarrow E$ 2
7	$\quad \neg Sb$	$\rightarrow E$ 5, 6
8	$Cb \rightarrow \neg Sb$	$\rightarrow I$ 3–7
9	$\forall z(Cz \rightarrow \neg Sz)$	$\forall I$ 8

$\forall x(Cx \rightarrow Fx), \forall y(Fy \rightarrow \neg Sy) \vdash \forall z(Cz \rightarrow \neg Sz)$

1	$\forall x(Cx \rightarrow Fx)$		
2	$\forall y(Fy \rightarrow \neg Sy)$		
3		Cb	Ass ($\rightarrow I$)
4		$Cb \rightarrow Fb$	$\forall E$ 1
5		Fb	$\rightarrow E$ 3, 4
6		$Fb \rightarrow \neg Sb$	$\rightarrow E$ 2
7		$\neg Sb$	$\rightarrow E$ 5, 6
8	$Cb \rightarrow \neg Sb$		$\rightarrow I$ 3–7
9	$\forall z(Cz \rightarrow \neg Sz)$		$\forall I$ 8

A Syllogism

All cats are fluffy. Nothing fluffy is scary. So no cats are scary.

$$\forall x(Cx \to Fx), \forall y(Fy \to \neg Sy) \quad \vdash \quad \forall z(Cz \to \neg Sz)$$

A Syllogism

All cats are fluffy. Nothing fluffy is scary. So no cats are scary.

$$\forall x(Cx \rightarrow Fx), \forall y(Fy \rightarrow \neg Sy) \quad \vdash \quad \forall z(Cz \rightarrow \neg Sz)$$

- So: the argument is an entailment

A Syllogism

All cats are fluffy. Nothing fluffy is scary. So no cats are scary.

$$\forall x(Cx \to Fx), \forall y(Fy \to \neg Sy) \quad \vdash \quad \forall z(Cz \to \neg Sz)$$

- So: the argument is an entailment
- So: the argument is valid

A Syllogism

Nothing finite is perfect. Everything imperfect has a creator. Therefore anything without a creator is infinite.

A Syllogism

Nothing finite is perfect. Everything imperfect has a creator. Therefore anything without a creator is infinite.

$$\begin{aligned} \text{domain} &: \text{all things} \\ Fx &: \underline{\quad}_x \text{ is finite} \\ Px &: \underline{\quad}_x \text{ is perfect} \\ Cx &: \underline{\quad}_x \text{ is created} \end{aligned}$$

A Syllogism

Nothing finite is perfect. Everything imperfect has a creator. Therefore anything without a creator is infinite.

$$\begin{aligned} \text{domain} &: \text{all things} \\ Fx &: \_\_\__x \text{ is finite} \\ Px &: \_\_\__x \text{ is perfect} \\ Cx &: \_\_\__x \text{ is created} \end{aligned}$$

$$\forall x(Fx \rightarrow \neg Px),\ \forall y(\neg Py \rightarrow Cy)\ \therefore\ \forall z(\neg Cz \rightarrow \neg Fz)$$

$\forall x(Fx \rightarrow \neg Px), \forall y(\neg Py \rightarrow Cy) \therefore \forall z(\neg Cz \rightarrow \neg Fz)$

1	$\forall x(Fx \rightarrow \neg Px)$
2	$\forall y(\neg Py \rightarrow Cy)$

$\forall x(Fx \rightarrow \neg Px), \forall y(\neg Py \rightarrow Cy) \therefore \forall z(\neg Cz \rightarrow \neg Fz)$

1	$\forall x(Fx \rightarrow \neg Px)$	
2	$\forall y(\neg Py \rightarrow Cy)$	
3	$\quad \neg Cd$	Ass ($\rightarrow I$)

$\forall x(Fx \rightarrow \neg Px), \forall y(\neg Py \rightarrow Cy) \therefore \forall z(\neg Cz \rightarrow \neg Fz)$

1	$\forall x(Fx \rightarrow \neg Px)$	
2	$\forall y(\neg Py \rightarrow Cy)$	
3	$\quad \neg Cd$	Ass $(\rightarrow I)$
4	$\quad \neg Pd \rightarrow Cd$	$\forall E$ 1

$\forall x(Fx \rightarrow \neg Px), \forall y(\neg Py \rightarrow Cy) \therefore \forall z(\neg Cz \rightarrow \neg Fz)$

1	$\forall x(Fx \rightarrow \neg Px)$	
2	$\forall y(\neg Py \rightarrow Cy)$	
3	$\neg Cd$	Ass ($\rightarrow I$)
4	$\neg Pd \rightarrow Cd$	$\forall E$ 1
5	$\neg\neg Pd$	MT 3, 4

$\forall x(Fx \rightarrow \neg Px), \forall y(\neg Py \rightarrow Cy) \therefore \forall z(\neg Cz \rightarrow \neg Fz)$

1	$\forall x(Fx \rightarrow \neg Px)$	
2	$\forall y(\neg Py \rightarrow Cy)$	
3	$\neg Cd$	Ass ($\rightarrow I$)
4	$\neg Pd \rightarrow Cd$	$\forall E$ 1
5	$\neg\neg Pd$	MT 3, 4
6	$Fd \rightarrow \neg Pd$	$\rightarrow E$ 2

$\forall x(Fx \rightarrow \neg Px), \forall y(\neg Py \rightarrow Cy) \therefore \forall z(\neg Cz \rightarrow \neg Fz)$

1	$\forall x(Fx \rightarrow \neg Px)$	
2	$\forall y(\neg Py \rightarrow Cy)$	
3	$\quad \neg Cd$	Ass $(\rightarrow I)$
4	$\quad \neg Pd \rightarrow Cd$	$\forall E$ 1
5	$\quad \neg\neg Pd$	MT 3, 4
6	$\quad Fd \rightarrow \neg Pd$	$\rightarrow E$ 2
7	$\quad \neg Fd$	MT 5, 6

$\forall x(Fx \to \neg Px),\ \forall y(\neg Py \to Cy)\ \therefore\ \forall z(\neg Cz \to \neg Fz)$

1	$\forall x(Fx \to \neg Px)$	
2	$\forall y(\neg Py \to Cy)$	
3	$\quad \neg Cd$	Ass ($\to I$)
4	$\quad \neg Pd \to Cd$	$\forall E$ 1
5	$\quad \neg\neg Pd$	MT 3, 4
6	$\quad Fd \to \neg Pd$	$\to E$ 2
7	$\quad \neg Fd$	MT 5, 6
8	$\neg Cd \to \neg Fd$	$\to I$ 3–7

$\forall x(Fx \rightarrow \neg Px),\ \forall y(\neg Py \rightarrow Cy)\ \therefore\ \forall z(\neg Cz \rightarrow \neg Fz)$

1	$\forall x(Fx \rightarrow \neg Px)$	
2	$\forall y(\neg Py \rightarrow Cy)$	
3	$\quad \neg Cd$	Ass $(\rightarrow I)$
4	$\quad \neg Pd \rightarrow Cd$	$\forall E$ 1
5	$\quad \neg\neg Pd$	MT 3, 4
6	$\quad Fd \rightarrow \neg Pd$	$\rightarrow E$ 2
7	$\quad \neg Fd$	MT 5, 6
8	$\neg Cd \rightarrow \neg Fd$	$\rightarrow I$ 3–7
9	$\forall z(\neg Cz \rightarrow \neg Fz)$	$\forall I$ 8

$\forall x(Fx \rightarrow \neg Px), \forall y(\neg Py \rightarrow Cy) \therefore \forall z(\neg Cz \rightarrow \neg Fz)$

1	$\forall x(Fx \rightarrow \neg Px)$	
2	$\forall y(\neg Py \rightarrow Cy)$	
3	$\quad \neg Cd$	Ass ($\rightarrow I$)
4	$\quad \neg Pd \rightarrow Cd$	$\forall E$ 1
5	$\quad \neg\neg Pd$	MT 3, 4
6	$\quad Fd \rightarrow \neg Pd$	$\rightarrow E$ 2
7	$\quad \neg Fd$	MT 5, 6
8	$\neg Cd \rightarrow \neg Fd$	$\rightarrow I$ 3–7
9	$\forall z(\neg Cz \rightarrow \neg Fz)$	$\forall I$ 8

A Syllogism

Nothing finite is perfect. Everything imperfect has a creator. Therefore anything without a creator is infinite.

$$\forall x(Fx \rightarrow \neg Px), \forall y(\neg Py \rightarrow Cy) \quad \vdash \quad \forall z(\neg Cz \rightarrow \neg Fz)$$

A Syllogism

Nothing finite is perfect. Everything imperfect has a creator. Therefore anything without a creator is infinite.

$$\forall x(Fx \rightarrow \neg Px),\ \forall y(\neg Py \rightarrow Cy) \quad \vdash \quad \forall z(\neg Cz \rightarrow \neg Fz)$$

- So: the argument is an entailment

A Syllogism

Nothing finite is perfect. Everything imperfect has a creator. Therefore anything without a creator is infinite.

$$\forall x(Fx \to \neg Px), \forall y(\neg Py \to Cy) \quad \vdash \quad \forall z(\neg Cz \to \neg Fz)$$

- So: the argument is an entailment
- So: the argument is valid

Existential Elimination

Existential Elimination

Existential Elimination ($\exists E$)

$$\begin{array}{l|l} & \exists x\, \mathscr{A}(\ldots x \ldots x \ldots) \\ & \quad \mathscr{A}(\ldots n \ldots n \ldots) \\ & \quad \vdots \\ & \quad \mathscr{C} \\ \triangleright & \mathscr{C} \end{array}$$

provided that: n does not appear outside of the sub-proof (in particular: n does not appear in $\mathscr{C}$)

Existential Elimination

Someone's committed a murder. I don't know who it is—but whoever it is, let's call them 'Jack the Ripper'. The murder was committed in Whitechapel, but murders are committed away from the murderer's home. So Jack the Ripper doesn't live in Whitechapel. So: someone who doesn't live in Whitechapel committed a murder.

Existential Elimination

1	$\forall y(My \rightarrow \neg Ly)$
2	$\exists x Mx$

Existential Elimination

1	$\forall y(My \to \neg Ly)$	
2	$\exists x Mx$	
3	$\quad Mj$	Ass ($\exists E$)

Existential Elimination

1	$\forall y(My \rightarrow \neg Ly)$	
2	$\exists x Mx$	
3	Mj	Ass ($\exists E$)
4	$Mj \rightarrow \neg Lj$	$\forall E$ 1

Existential Elimination

1	$\forall y(My \to \neg Ly)$	
2	$\exists x Mx$	
3	Mj	Ass ($\exists E$)
4	$Mj \to \neg Lj$	$\forall E$ 1
5	$\neg Lj$	$\to E$ 3, 4

Existential Elimination

1	$\forall y(My \rightarrow \neg Ly)$	
2	$\exists x Mx$	
3	Mj	Ass ($\exists E$)
4	$Mj \rightarrow \neg Lj$	$\forall E$ 1
5	$\neg Lj$	$\rightarrow E$ 3, 4
6	$Mj \wedge \neg Lj$	$\wedge I$ 3, 5

Existential Elimination

1	$\forall y(My \to \neg Ly)$	
2	$\exists x Mx$	
3	Mj	Ass ($\exists E$)
4	$Mj \to \neg Lj$	$\forall E$ 1
5	$\neg Lj$	$\to E$ 3, 4
6	$Mj \wedge \neg Lj$	$\wedge I$ 3, 5
7	$\exists x(Mx \wedge \neg Lx)$	$\exists I$ 6

Existential Elimination

1	$\forall y(My \rightarrow \neg Ly)$	
2	$\exists x Mx$	
3	$\quad Mj$	Ass $(\exists E)$
4	$\quad Mj \rightarrow \neg Lj$	$\forall E$ 1
5	$\quad \neg Lj$	$\rightarrow E$ 3, 4
6	$\quad Mj \wedge \neg Lj$	$\wedge I$ 3, 5
7	$\quad \exists x(Mx \wedge \neg Lx)$	$\exists I$ 6
8	$\exists x(Mx \wedge \neg Lx)$	$\exists E$ 2, 3–7

Existential Elimination

1	$\forall y(My \to \neg Ly)$	
2	$\exists x Mx$	
3	$\quad Mj$	Ass ($\exists E$)
4	$\quad Mj \to \neg Lj$	$\forall E$ 1
5	$\quad \neg Lj$	$\to E$ 3, 4
6	$\quad Mj \wedge \neg Lj$	$\wedge I$ 3, 5
7	$\quad \exists x(Mx \wedge \neg Lx)$	$\exists I$ 6
8	$\exists x(Mx \wedge \neg Lx)$	$\exists E$ 2, 3–7

Existential Elimination

1	$\forall y(My \rightarrow \neg Ly)$	
2	$\exists xMx$	
3	Mp	Ass ($\exists E$)
4	$Mp \rightarrow \neg Lp$	$\forall E$ 1
5	$\neg Lp$	$\rightarrow E$ 3, 4
6	$Mp \wedge \neg Lp$	$\wedge I$ 3, 5
7	$\exists x(Mx \wedge \neg Lx)$	$\exists I$ 6
8	$\exists x(Mx \wedge \neg Lx)$	$\exists E$ 2, 3–7

Existential Elimination

1	$\forall y(My \rightarrow \neg Ly)$	
2	$\exists x Mx$	
3	Ma	Ass ($\exists E$)
4	$Ma \rightarrow \neg La$	$\forall E$ 1
5	$\neg La$	$\rightarrow E$ 3, 4
6	$Ma \wedge \neg La$	$\wedge I$ 3, 5
7	$\exists x(Mx \wedge \neg Lx)$	$\exists I$ 6
8	$\exists x(Mx \wedge \neg Lx)$	$\exists E$ 2, 3–7

Existential Elimination

1	$\forall y(My \rightarrow \neg Ly)$	
2	$\exists x Mx$	
3	Mc	Ass ($\exists E$)
4	$Mc \rightarrow \neg Lc$	$\forall E$ 1
5	$\neg Lc$	$\rightarrow E$ 3, 4
6	$Mc \wedge \neg Lc$	$\wedge I$ 3, 5
7	$\exists x(Mx \wedge \neg Lx)$	$\exists I$ 6
8	$\exists x(Mx \wedge \neg Lx)$	$\exists E$ 2, 3–7

Existential Elimination

1	$\forall y\, Hey$	
2	$\exists x Fx$	
3	$\quad Fe$	Ass ($\exists E$)
4	$\quad Hee$	$\forall E$ 1
5	$\quad Fe \wedge Hee$	$\wedge I$ 3, 4
6	$\quad \exists y(Fy \wedge Hyy)$	$\exists I$ 5

Existential Elimination

1	$\forall y\, Hey$	
2	$\exists x Fx$	
3	Fe	Ass ($\exists E$)
4	Hee	$\forall E$ 1
5	$Fe \wedge Hee$	$\wedge I$ 3, 4
6	$\exists y(Fy \wedge Hyy)$	$\exists I$ 5
7	$\exists y(Fy \wedge Hyy)$	$\exists E$ 2, 3–6

Existential Elimination

1	$\forall y\, Hey$	
2	$\exists x Fx$	
3	Fe	Ass ($\exists E$)
4	Hee	$\forall E$ 1
5	$Fe \wedge Hee$	$\wedge I$ 3, 4
6	$\exists y(Fy \wedge Hyy)$	$\exists I$ 5
7	$\exists y(Fy \wedge Hyy)$	$\exists E$ 2, 3–6 $\longleftarrow$ **MISTAKE!**

Existential Elimination

1	$\forall y\, Hey$	
2	$\exists x Fx$	
3	$\quad Fe$	Ass ($\exists E$)
4	$\quad Hee$	$\forall E$ 1
5	$\quad Fe \wedge Hee$	$\wedge I$ 3, 4
6	$\quad \exists y(Fy \wedge Hyy)$	$\exists I$ 5
7	$\exists y(Fy \wedge Hyy)$	$\exists E$ 2, 3–6 $\longleftarrow$ **MISTAKE!**

- '*e*' appears outside of the sub-proof (on line 1).

Existential Elimination

1	$\exists x Rxx$	
2	Rpp	Ass ($\exists E$)
3	$\exists y Rpy$	$\exists I$ 2

Existential Elimination

1	$\exists x Rxx$	
2	$\quad Rpp$	Ass $(\exists E)$
3	$\quad \exists y Rpy$	$\exists I$ 2
4	$\exists y Rpy$	$\exists E$ 1, 2–3

Existential Elimination

1	$\exists x Rxx$	
2	$\quad Rpp$	Ass ($\exists E$)
3	$\quad \exists y Rpy$	$\exists I$ 2
4	$\exists y Rpy$	$\exists E$ 1, 2–3 $\longleftarrow$ **MISTAKE!**

Existential Elimination

1	$\exists x Rxx$	
2	Rpp	Ass ($\exists E$)
3	$\exists y Rpy$	$\exists I$ 2
4	$\exists y Rpy$	$\exists E$ 1, 2–3 $\longleftarrow$ **MISTAKE!**

- ‘p’ appears outside of the sub-proof (on line 4)

A Syllogism

Some pundits argue in bad faith. No one who argues in bad faith is worth arguing with. So some pundits are not worth arguing with.

A Syllogism

Some pundits argue in bad faith. No one who argues in bad faith is worth arguing with. So some pundits are not worth arguing with.

$$\begin{aligned} \text{domain} &: \text{all people} \\ Px &: \underline{\quad}_x \text{ is pundit} \\ Ax &: \underline{\quad}_x \text{ argues in bad faith} \\ Wx &: \underline{\quad}_x \text{ is worth arguing with} \end{aligned}$$

A Syllogism

Some pundits argue in bad faith. No one who argues in bad faith is worth arguing with. So some pundits are not worth arguing with.

$$\begin{aligned} \text{domain} &: \text{all people} \\ Px &: \underline{\quad}_x \text{ is pundit} \\ Ax &: \underline{\quad}_x \text{ argues in bad faith} \\ Wx &: \underline{\quad}_x \text{ is worth arguing with} \end{aligned}$$

$$\exists x(Px \wedge Ax),\ \forall y(Ay \rightarrow \neg Wy)\ \therefore\ \exists z(Pz \wedge \neg Wz)$$

1	$\exists x(Px \land Ax)$
2	$\forall y(Ay \to \neg Wy)$

1	$\exists x(Px \wedge Ax)$	
2	$\forall y(Ay \rightarrow \neg Wy)$	
3	$Pm \wedge Am$	Ass ($\exists E$)

1	$\exists x(Px \land Ax)$	
2	$\forall y(Ay \rightarrow \neg Wy)$	
3	$\quad Pm \land Am$	Ass ($\exists E$)
4	$\quad Am$	$\land E$ 3

1	$\exists x(Px \land Ax)$	
2	$\forall y(Ay \to \neg Wy)$	
3	$Pm \land Am$	Ass ($\exists E$)
4	Am	$\land E$ 3
5	$Am \to \neg Wm$	$\forall E$ 2

1	$\exists x(Px \wedge Ax)$	
2	$\forall y(Ay \rightarrow \neg Wy)$	
3	$Pm \wedge Am$	Ass ($\exists E$)
4	Am	$\wedge E$ 3
5	$Am \rightarrow \neg Wm$	$\forall E$ 2
6	$\neg Wm$	$\rightarrow E$ 4, 5

1	$\exists x(Px \land Ax)$	
2	$\forall y(Ay \rightarrow \neg Wy)$	
3	$\quad Pm \land Am$	Ass ($\exists E$)
4	$\quad Am$	$\land E$ 3
5	$\quad Am \rightarrow \neg Wm$	$\forall E$ 2
6	$\quad \neg Wm$	$\rightarrow E$ 4, 5
7	$\quad Pm$	$\land E$ 3

1	$\exists x(Px \wedge Ax)$	
2	$\forall y(Ay \rightarrow \neg Wy)$	
3	$Pm \wedge Am$	Ass ($\exists E$)
4	Am	$\wedge E$ 3
5	$Am \rightarrow \neg Wm$	$\forall E$ 2
6	$\neg Wm$	$\rightarrow E$ 4, 5
7	Pm	$\wedge E$ 3
8	$Pm \wedge \neg Wm$	$\wedge I$ 6, 7

1	$\exists x(Px \land Ax)$	
2	$\forall y(Ay \rightarrow \neg Wy)$	
3	$\quad Pm \land Am$	Ass ($\exists E$)
4	$\quad Am$	$\land E$ 3
5	$\quad Am \rightarrow \neg Wm$	$\forall E$ 2
6	$\quad \neg Wm$	$\rightarrow E$ 4, 5
7	$\quad Pm$	$\land E$ 3
8	$\quad Pm \land \neg Wm$	$\land I$ 6, 7
9	$\quad \exists z(Pz \land \neg Wz)$	$\exists I$ 8

1	$\exists x(Px \land Ax)$	
2	$\forall y(Ay \rightarrow \neg Wy)$	
3	$Pm \land Am$	Ass ($\exists E$)
4	Am	$\land E$ 3
5	$Am \rightarrow \neg Wm$	$\forall E$ 2
6	$\neg Wm$	$\rightarrow E$ 4, 5
7	Pm	$\land E$ 3
8	$Pm \land \neg Wm$	$\land I$ 6, 7
9	$\exists z(Pz \land \neg Wz)$	$\exists I$ 8
10	$\exists z(Pz \land \neg Wz)$	$\exists E$ 1, 3–9

1	$\exists x(Px \wedge Ax)$	
2	$\forall y(Ay \rightarrow \neg Wy)$	
3	$\quad Pm \wedge Am$	Ass ($\exists E$)
4	$\quad Am$	$\wedge E$ 3
5	$\quad Am \rightarrow \neg Wm$	$\forall E$ 2
6	$\quad \neg Wm$	$\rightarrow E$ 4, 5
7	$\quad Pm$	$\wedge E$ 3
8	$\quad Pm \wedge \neg Wm$	$\wedge I$ 6, 7
9	$\quad \exists z(Pz \wedge \neg Wz)$	$\exists I$ 8
10	$\exists z(Pz \wedge \neg Wz)$	$\exists E$ 1, 3–9

A Syllogism

Some pundits argue in bad faith. No one who argues in bad faith is worth arguing with. So some pundits are not worth arguing with.

$$\exists x(Px \land Ax), \forall y(Ay \to \neg Wy) \quad \vdash \quad \exists z(Pz \land \neg Wz)$$

A Syllogism

Some pundits argue in bad faith. No one who argues in bad faith is worth arguing with. So some pundits are not worth arguing with.

$$\exists x(Px \wedge Ax), \forall y(Ay \rightarrow \neg Wy) \quad \vdash \quad \exists z(Pz \wedge \neg Wz)$$

- So: the argument is an entailment

A Syllogism

Some pundits argue in bad faith. No one who argues in bad faith is worth arguing with. So some pundits are not worth arguing with.

$$\exists x(Px \wedge Ax), \forall y(Ay \rightarrow \neg Wy) \quad \vdash \quad \exists z(Pz \wedge \neg Wz)$$

- So: the argument is an entailment
- So: the argument is valid

An Argument

Someone loves everyone. So everyone has someone who loves them.

An Argument

Someone loves everyone. So everyone has someone who loves them.

$$\exists x \forall y\, Lxy \therefore \forall y \exists x\, Lxy$$

$\exists x \forall y Lxy \vdash \forall y \exists x Lxy$

1	$\exists x \forall y Lxy$

$\exists x \forall y Lxy \vdash \forall y \exists x Lxy$

1	$\exists x \forall y Lxy$	
2	$\forall y Lay$	Ass ($\exists E$)

$\exists x \forall y Lxy \vdash \forall y \exists x Lxy$

1	$\exists x \forall y Lxy$	
2	$\forall y Lay$	Ass ($\exists E$)
3	Lab	$\forall E$ 2

$\exists x \forall y Lxy \vdash \forall y \exists x Lxy$

1	$\exists x \forall y Lxy$	
2	$\forall y Lay$	Ass ($\exists E$)
3	Lab	$\forall E$ 2
4	$\exists x Lxb$	$\exists I$ 3

$\exists x \forall y Lxy \vdash \forall y \exists x Lxy$

1	$\exists x \forall y Lxy$	
2	$\quad \forall y Lay$	Ass ($\exists E$)
3	$\quad Lab$	$\forall E$ 2
4	$\quad \exists x Lxb$	$\exists I$ 3
5	$\quad \forall y \exists x Lxy$	$\forall I$ 4

$\exists x \forall y Lxy \vdash \forall y \exists x Lxy$

1	$\exists x \forall y Lxy$	
2	$\forall y Lay$	Ass ($\exists E$)
3	Lab	$\forall E$ 2
4	$\exists x Lxb$	$\exists I$ 3
5	$\forall y \exists x Lxy$	$\forall I$ 4
6	$\forall y \exists x Lxy$	$\exists E$ 1, 2–5

$\exists x \forall y Lxy \vdash \forall y \exists x Lxy$

1	$\exists x \forall y Lxy$	
2	$\forall y Lay$	Ass ($\exists E$)
3	Lab	$\forall E$ 2
4	$\exists x Lxb$	$\exists I$ 3
5	$\forall y \exists x Lxy$	$\forall I$ 4
6	$\forall y \exists x Lxy$	$\exists E$ 1, 2–5

Universal Introduction

Universal Introduction ($\forall I$)

$$\mathscr{A}(\ldots n \ldots n \ldots)$$

$$\triangleright \quad \forall x\, \mathscr{A}(\ldots x \ldots x \ldots)$$

provided that: n *does not appear in any open assumption*

Universal Elimination

Universal Elimination ($\forall E$)

$$\forall x\, \mathscr{A}(\ldots x \ldots x \ldots)$$

$$\triangleright \quad \mathscr{A}(\ldots n \ldots n \ldots)$$

Existential Introduction

Existential Introduction ($\exists I$)

$$\mathscr{A}(\ldots n \ldots n \ldots)$$

$$\triangleright \quad \exists x\, \mathscr{A}(\ldots n \ldots x \ldots)$$

Existential Elimination

Existential Elimination ($\exists E$)

$$
\begin{array}{l|l}
 & \exists x\, \mathscr{A}(\ldots x \ldots x \ldots) \\
 & \quad \mathscr{A}(\ldots n \ldots n \ldots) \\
 & \quad \vdots \\
 & \quad \mathscr{C} \\
\triangleright & \mathscr{C}
\end{array}
$$

provided that: n does not appear outside of the sub-proof (in particular: n does not appear in $\mathscr{C}$)

Derived Rules

An Argument

Not everyone is fun. So, someone is not fun.

An Argument

Not everyone is fun. So, someone is not fun.

$$\neg\forall x Fx \therefore \exists x \neg Fx$$

$\neg \forall x\, Fx \vdash \exists x\, \neg Fx$

1 $\neg \forall x\, Fx$

$\neg\forall x\, Fx \;\vdash\; \exists x\, \neg Fx$

1	$\neg\forall x\, Fx$	
2	$\neg\exists x \neg Fx$	Ass ($\neg E$)

$\neg \forall x\, Fx \vdash \exists x \neg Fx$

1	$\neg\forall x\, Fx$	
2	$\neg\exists x \neg Fx$	Ass ($\neg E$)
3	$\neg Fa$	Ass ($\neg E$)

$\neg \forall x\, Fx \vdash \exists x \neg Fx$

1	$\neg \forall x\, Fx$	
2	$\quad \neg \exists x \neg Fx$	Ass $(\neg E)$
3	$\quad\quad \neg Fa$	Ass $(\neg E)$
4	$\quad\quad \exists x \neg Fx$	$\exists I$ 3

$\neg\forall x\, Fx \vdash \exists x \neg Fx$

1	$\neg\forall x\, Fx$	
2	$\neg\exists x \neg Fx$	Ass ($\neg E$)
3	$\neg Fa$	Ass ($\neg E$)
4	$\exists x \neg Fx$	$\exists I$ 3
5	$\bot$	$\bot I$ 2, 4

$\neg \forall x\, Fx \ \vdash\ \exists x\, \neg Fx$

1	$\neg\forall x\, Fx$	
2	$\neg\exists x \neg Fx$	Ass ($\neg E$)
3	$\neg Fa$	Ass ($\neg E$)
4	$\exists x \neg Fx$	$\exists I$ 3
5	$\bot$	$\bot I$ 2, 4
6	Fa	$\neg I$ 3–5

$\neg \forall x\, Fx \vdash \exists x \neg Fx$

1	$\neg \forall x\, Fx$	
2	$\neg \exists x \neg Fx$	Ass ($\neg E$)
3	$\neg Fa$	Ass ($\neg E$)
4	$\exists x \neg Fx$	$\exists I$ 3
5	$\bot$	$\bot I$ 2, 4
6	Fa	$\neg I$ 3–5
7	$\forall x Fx$	$\forall I$ 6

$\neg\forall x\,Fx \vdash \exists x\neg Fx$

1	$\neg\forall x\,Fx$	
2	$\quad\neg\exists x\neg Fx$	Ass $(\neg E)$
3	$\quad\quad\neg Fa$	Ass $(\neg E)$
4	$\quad\quad\exists x\neg Fx$	$\exists I$ 3
5	$\quad\quad\bot$	$\bot I$ 2, 4
6	$\quad Fa$	$\neg I$ 3–5
7	$\quad\forall xFx$	$\forall I$ 6
8	$\quad\bot$	$\bot I$ 1, 7

$\neg \forall x\, Fx \vdash \exists x \neg Fx$

1	$\neg \forall x\, Fx$	
2	$\neg \exists x \neg Fx$	Ass ($\neg E$)
3	$\neg Fa$	Ass ($\neg E$)
4	$\exists x \neg Fx$	$\exists I$ 3
5	$\bot$	$\bot I$ 2, 4
6	Fa	$\neg I$ 3–5
7	$\forall x Fx$	$\forall I$ 6
8	$\bot$	$\bot I$ 1, 7
9	$\exists x \neg Fx$	$\neg E$ 2–8

$\neg \forall x\, Fx \vdash \exists x \neg Fx$

1	$\neg \forall x\, Fx$	
2	$\quad \neg \exists x \neg Fx$	Ass ($\neg E$)
3	$\quad\quad \neg Fa$	Ass ($\neg E$)
4	$\quad\quad \exists x \neg Fx$	$\exists I$ 3
5	$\quad\quad \bot$	$\bot I$ 2, 4
6	$\quad Fa$	$\neg I$ 3–5
7	$\quad \forall x Fx$	$\forall I$ 6
8	$\quad \bot$	$\bot I$ 1, 7
9	$\exists x \neg Fx$	$\neg E$ 2–8

Change of Quantifiers (CQ)

1 $\neg\forall x\mathscr{A}(\ldots x\ldots x\ldots)$

Change of Quantifiers (CQ)

1	$\neg\forall x \mathscr{A}(\ldots x \ldots x \ldots)$	
2	$\quad \neg\exists x \neg\mathscr{A}(\ldots x \ldots x \ldots)$	Ass ($\neg E$)

Change of Quantifiers (CQ)

1	$\neg \forall x \mathscr{A}(\ldots x \ldots x \ldots)$	
2	$\neg \exists x \neg \mathscr{A}(\ldots x \ldots x \ldots)$	Ass ($\neg E$)
3	$\neg \mathscr{A}(\ldots n \ldots n \ldots)$	Ass ($\neg E$)

Change of Quantifiers (CQ)

1	$\neg\forall x \mathscr{A}(\ldots x \ldots x \ldots)$	
2	$\neg\exists x \neg\mathscr{A}(\ldots x \ldots x \ldots)$	Ass ($\neg E$)
3	$\neg\mathscr{A}(\ldots n \ldots n \ldots)$	Ass ($\neg E$)
4	$\exists x \neg\mathscr{A}(\ldots x \ldots x \ldots)$	$\exists I$ 3

Change of Quantifiers (CQ)

1	$\neg\forall x \mathscr{A}(\ldots x \ldots x \ldots)$	
2	$\quad \neg\exists x \neg\mathscr{A}(\ldots x \ldots x \ldots)$	Ass ($\neg E$)
3	$\quad\quad \neg\mathscr{A}(\ldots n \ldots n \ldots)$	Ass ($\neg E$)
4	$\quad\quad \exists x \neg\mathscr{A}(\ldots x \ldots x \ldots)$	$\exists I$ 3
5	$\quad\quad \bot$	$\bot I$ 2, 4

Change of Quantifiers (CQ)

1	$\neg\forall x \mathscr{A}(\ldots x \ldots x \ldots)$	
2	$\quad \neg\exists x \neg\mathscr{A}(\ldots x \ldots x \ldots)$	Ass ($\neg E$)
3	$\quad\quad \neg\mathscr{A}(\ldots n \ldots n \ldots)$	Ass ($\neg E$)
4	$\quad\quad \exists x \neg\mathscr{A}(\ldots x \ldots x \ldots)$	$\exists I$ 3
5	$\quad\quad \bot$	$\bot I$ 2, 4
6	$\quad \mathscr{A}(\ldots n \ldots n \ldots)$	$\neg E$ 3–5

Change of Quantifiers (CQ)

1	$\neg\forall x \mathscr{A}(\ldots x \ldots x \ldots)$	
2	$\neg\exists x \neg\mathscr{A}(\ldots x \ldots x \ldots)$	Ass ($\neg E$)
3	$\neg\mathscr{A}(\ldots n \ldots n \ldots)$	Ass ($\neg E$)
4	$\exists x \neg\mathscr{A}(\ldots x \ldots x \ldots)$	$\exists I$ 3
5	$\bot$	$\bot I$ 2, 4
6	$\mathscr{A}(\ldots n \ldots n \ldots)$	$\neg E$ 3–5
7	$\forall x \mathscr{A}(\ldots x \ldots x \ldots)$	$\forall I$ 6

Change of Quantifiers (CQ)

1	$\neg\forall x \mathscr{A}(\ldots x \ldots x \ldots)$	
2	$\quad \neg\exists x \neg\mathscr{A}(\ldots x \ldots x \ldots)$	Ass ($\neg E$)
3	$\quad\quad \neg\mathscr{A}(\ldots n \ldots n \ldots)$	Ass ($\neg E$)
4	$\quad\quad \exists x \neg\mathscr{A}(\ldots x \ldots x \ldots)$	$\exists I$ 3
5	$\quad\quad \bot$	$\bot I$ 2, 4
6	$\quad \mathscr{A}(\ldots n \ldots n \ldots)$	$\neg E$ 3–5
7	$\quad \forall x \mathscr{A}(\ldots x \ldots x \ldots)$	$\forall I$ 6
8	$\quad \bot$	$\bot I$ 1, 7

Change of Quantifiers (CQ)

1	$\neg\forall x \mathscr{A}(\ldots x \ldots x \ldots)$	
2	$\neg\exists x \neg\mathscr{A}(\ldots x \ldots x \ldots)$	Ass ($\neg E$)
3	$\neg\mathscr{A}(\ldots n \ldots n \ldots)$	Ass ($\neg E$)
4	$\exists x \neg\mathscr{A}(\ldots x \ldots x \ldots)$	$\exists I$ 3
5	$\bot$	$\bot I$ 2, 4
6	$\mathscr{A}(\ldots n \ldots n \ldots)$	$\neg E$ 3–5
7	$\forall x \mathscr{A}(\ldots x \ldots x \ldots)$	$\forall I$ 6
8	$\bot$	$\bot I$ 1, 7
9	$\exists x \neg\mathscr{A}(\ldots x \ldots x \ldots)$	$\neg E$ 2-8

Change of Quantifiers (CQ)

1	$\neg\forall x \mathscr{A}(\ldots x \ldots x \ldots)$	
2	$\mid\ \neg\exists x \neg\mathscr{A}(\ldots x \ldots x \ldots)$	Ass ($\neg E$)
3	$\mid\ \mid\ \neg\mathscr{A}(\ldots n \ldots n \ldots)$	Ass ($\neg E$)
4	$\mid\ \mid\ \exists x \neg\mathscr{A}(\ldots x \ldots x \ldots)$	$\exists I$ 3
5	$\mid\ \mid\ \bot$	$\bot I$ 2, 4
6	$\mid\ \mathscr{A}(\ldots n \ldots n \ldots)$	$\neg E$ 3–5
7	$\mid\ \forall x \mathscr{A}(\ldots x \ldots x \ldots)$	$\forall I$ 6
8	$\mid\ \bot$	$\bot I$ 1, 7
9	$\exists x \neg\mathscr{A}(\ldots x \ldots x \ldots)$	$\neg E$ 2-8

Change of Quantifiers (CQ)

Change of Quantifiers (CQ)

$$\neg\forall x \mathscr{A} \quad \triangleright \quad \exists x \neg \mathscr{A}$$

Change of Quantifiers (CQ)

1 $\exists x \neg \mathscr{A}(\ldots x \ldots x \ldots)$

Change of Quantifiers (CQ)

1	$\exists x \neg \mathscr{A}(\ldots x \ldots x \ldots)$	
2	$\neg \mathscr{A}(\ldots n \ldots n \ldots)$	Ass ($\exists E$)

Change of Quantifiers (CQ)

1	$\exists x \neg \mathscr{A}(\ldots x \ldots x \ldots)$	
2	$\quad \neg \mathscr{A}(\ldots n \ldots n \ldots)$	Ass ($\exists E$)
3	$\quad\quad \forall x \mathscr{A}(\ldots x \ldots x \ldots)$	Ass ($\neg I$)

Change of Quantifiers (CQ)

1	$\exists x \neg \mathscr{A}(\ldots x \ldots x \ldots)$	
2	$\neg \mathscr{A}(\ldots n \ldots n \ldots)$	Ass ($\exists E$)
3	$\forall x \mathscr{A}(\ldots x \ldots x \ldots)$	Ass ($\neg I$)
4	$\mathscr{A}(\ldots n \ldots n \ldots)$	$\forall E$ 3

Change of Quantifiers (CQ)

1	$\exists x \neg \mathscr{A}(\ldots x \ldots x \ldots)$	
2	$\neg \mathscr{A}(\ldots n \ldots n \ldots)$	Ass ($\exists E$)
3	$\forall x \mathscr{A}(\ldots x \ldots x \ldots)$	Ass ($\neg I$)
4	$\mathscr{A}(\ldots n \ldots n \ldots)$	$\forall E$ 3
5	$\bot$	$\bot I$ 2, 4

Change of Quantifiers (CQ)

1	$\exists x \neg \mathscr{A}(\ldots x \ldots x \ldots)$	
2	$\quad \neg \mathscr{A}(\ldots n \ldots n \ldots)$	Ass ($\exists E$)
3	$\quad\quad \forall x \mathscr{A}(\ldots x \ldots x \ldots)$	Ass ($\neg I$)
4	$\quad\quad \mathscr{A}(\ldots n \ldots n \ldots)$	$\forall E$ 3
5	$\quad\quad \bot$	$\bot I$ 2, 4
6	$\quad \neg \forall x \mathscr{A}(\ldots x \ldots x \ldots)$	$\neg I$ 3–5

Change of Quantifiers (CQ)

1	$\exists x \neg \mathscr{A}(\ldots x \ldots x \ldots)$	
2	$\neg \mathscr{A}(\ldots n \ldots n \ldots)$	Ass ($\exists E$)
3	$\forall x \mathscr{A}(\ldots x \ldots x \ldots)$	Ass ($\neg I$)
4	$\mathscr{A}(\ldots n \ldots n \ldots)$	$\forall E$ 3
5	$\bot$	$\bot I$ 2, 4
6	$\neg \forall x \mathscr{A}(\ldots x \ldots x \ldots)$	$\neg I$ 3–5
7	$\neg \forall x \mathscr{A}(\ldots x \ldots x \ldots)$	$\exists E$ 1, 2–6

Change of Quantifiers (CQ)

1	$\exists x \neg \mathscr{A}(\ldots x \ldots x \ldots)$	
2	$\quad \neg \mathscr{A}(\ldots n \ldots n \ldots)$	Ass ($\exists E$)
3	$\quad\quad \forall x \mathscr{A}(\ldots x \ldots x \ldots)$	Ass ($\neg I$)
4	$\quad\quad \mathscr{A}(\ldots n \ldots n \ldots)$	$\forall E$ 3
5	$\quad\quad \bot$	$\bot I$ 2, 4
6	$\quad \neg \forall x \mathscr{A}(\ldots x \ldots x \ldots)$	$\neg I$ 3–5
7	$\neg \forall x \mathscr{A}(\ldots x \ldots x \ldots)$	$\exists E$ 1, 2–6

Change of Quantifiers (CQ)

Change of Quantifiers (CQ)

$$\neg\forall x \mathscr{A} \qquad \triangleleft \; \triangleright \qquad \exists x \neg\mathscr{A}$$

Change of Quantifiers (CQ)

1	$\neg\exists x \mathscr{A}(\ldots x \ldots x \ldots)$	
2	$\mathscr{A}(\ldots n \ldots n \ldots)$	Ass ($\neg I$)
3	$\exists x \mathscr{A}(\ldots x \ldots x \ldots)$	$\exists I$ 2
4	$\bot$	$\bot I$ 1, 3
5	$\neg\mathscr{A}(\ldots n \ldots n \ldots)$	$\neg I$ 2–4
6	$\forall x \neg\mathscr{A}(\ldots x \ldots x \ldots)$	$\forall I$ 5

Change of Quantifiers (CQ)

Change of Quantifiers (CQ)

$\neg\forall x\mathscr{A}$	$\triangleleft\ \triangleright$	$\exists x\neg\mathscr{A}$
$\neg\exists x\mathscr{A}$	$\triangleright$	$\forall x\neg\mathscr{A}$

Change of Quantifiers (CQ)

1	$\forall x \neg \mathscr{A}(\ldots x \ldots x \ldots)$	
2	$\quad \exists x \mathscr{A}(\ldots x \ldots x \ldots)$	Ass ($\neg I$)
3	$\quad\quad \mathscr{A}(\ldots n \ldots n \ldots)$	Ass ($\exists E$)
4	$\quad\quad \neg \mathscr{A}(\ldots n \ldots n \ldots)$	$\forall E$ 1
5	$\quad\quad \bot$	$\bot I$ 3, 4
6	$\quad \bot$	$\exists E$ 2, 3–5
7	$\neg \exists x \mathscr{A}(\ldots x \ldots x \ldots)$	$\neg I$ 2–6

Change of Quantifiers (CQ)

Change of Quantifiers (CQ)

$\neg\forall x \mathscr{A}$	$\triangleleft \ \triangleright$	$\exists x \neg \mathscr{A}$
$\neg\exists x \mathscr{A}$	$\triangleleft \ \triangleright$	$\forall x \neg \mathscr{A}$