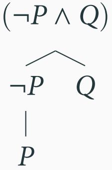
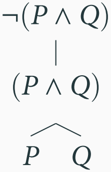
Sentential Logic

Syntax and Semantics

PHIL 500





Outline

The Language SL

- Syntax for SL
- Semantics for SL

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The Plan

- Construct an artificial language, 'SL', within which we can be incredibly precise about which arguments are deductively valid and which are deductively invalid.

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- Provide a method for translating statements of English into SL and statements of SL into English
- The advantage: we can theorize about relations of deductive validity without having to worry about the ambiguity of English

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  - a way to interpret the *meaning* of every grammatical expression of the language
    - *e.g.*, a dictionary entry for every word of English and rules for constructing the meaning of sentences out of the meanings of words

Languages in General

SYNTAX — { 1. Vocabulary / 2. Grammar }

SEMANTICS — 3. Meaning

1. Vocabulary
2. Grammar
3. Meaning

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The Language SL

Syntax for SL

Semantics for SL

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  1. An infinite number of statement letters:

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- Nothing else is included in the vocabulary of SL.

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$$(((\,)A_{23} \wedge \wedge \rightarrow\rightarrow Z$$

$$P \rightarrow (Q) \supset \wedge(\,))$$

$$(P \rightarrow (Q \rightarrow (R \rightarrow (S \rightarrow T))))$$

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  - –) Nothing else is a sentence.

Meta-variables

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- '$\mathscr{A}$' and '$\mathscr{B}$' do not appear in the vocabulary of SL.
- They are *metavariables*—variables whose potential values are sentences of SL.

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- To show that ‘$(\neg(P \vee Q) \rightarrow R)$’ is a sentence:
    a) ‘P’ is a sentence [from (*SL*)]

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  e) 'R' is a sentence [from (SL)]
  f) So, '$(\neg(P \vee Q) \rightarrow R)$' is a sentence [from (d), (e), and ($\rightarrow$)]

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- $((X \land Y \land Z) \to W)$?

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- $[(A \vee B) \wedge (A \leftrightarrow S)] \rightarrow [(R \wedge T) \rightarrow \neg Q]$

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  b) So, '$\neg P$' is a sentence [from (a) and ($\neg$)]
  c) 'Q' is a sentence [from (SL)]
  d) So, '$(\neg P \land Q)$' is a sentence [from (b), (c), and ($\land$)]
- So, the main operator is '$\land$'

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- What if we had applied the rule ($\wedge$) first, and then applied the rule ($\neg$)?

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- What if we had applied the rule (∧) first, and then applied the rule (¬)?
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  d) So, '$\neg(P \wedge Q)$' is a sentence [from (c) and ($\neg$)]
- '$\neg(P \wedge Q)$' is not the same sentence as '$(\neg P \wedge Q)$'

Subsentences

- '$\mathscr{B}$' is a *subsentence* of '$\mathscr{A}$' if and only if, in the course of building up '$\mathscr{A}$' by applying the rules for sentences, '$\mathscr{B}$' appears on a line before '$\mathscr{A}$'.

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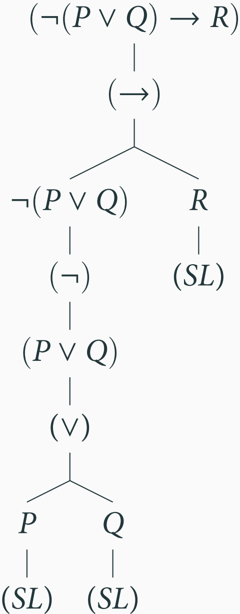
    - '$\neg P$' is *not* a subsentence of '$\neg(P \wedge Q)$'

Syntactic Structure



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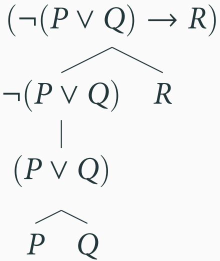

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▹ The scope of '$\neg$' in

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$$\neg[(A \leftrightarrow B) \wedge (S \vee \neg T)]$$

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- Parenthases just make syntactic structure explicit. They do not make any contribution to meaning beyond that.
- So: we must say what the meanings of the statements letters are and what the meanings of the logical operators are

The Meaning of the Statement Letters

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- The statement letter is true if and only if the statement in English is true.

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$\mathscr{A}$	$\neg\mathscr{A}$
T	F
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- Note: '$\mathscr{A}$' is not a sentence of SL
    - we are using '$\mathscr{A}$' and '$\mathscr{B}$' as variables ranging over the sentences of SL

The Meaning of ‘$\wedge$’

- A sentence whose main operator is ‘$\wedge$’ is known as a *conjunction*.

The Meaning of '∧'

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$\mathscr{A}$	$\mathscr{B}$	$\mathscr{A} \wedge \mathscr{B}$
T	T	
T	F	
F	T	
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The Meaning of '∨'

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T	T	
T	F	
F	T	
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$\mathscr{A}$	$\mathscr{B}$	$\mathscr{A} \vee \mathscr{B}$
T	T	T
T	F	T
F	T	T
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T	T	T
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- Note: this is the only binary operator which is not symmetric.
    - '$\mathscr{A} \rightarrow \mathscr{B}$' *does not* have the same meaning as '$\mathscr{B} \rightarrow \mathscr{A}$'

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F	T	
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$\mathscr{A}$	$\mathscr{B}$	$\mathscr{A} \rightarrow \mathscr{B}$
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T	F	
F	T	
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Determining the truth-value of a sentence of SL

- Suppose '*P*' is true and '*Q*' is false

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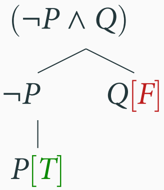

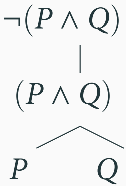

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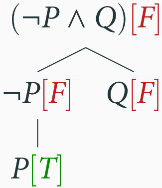

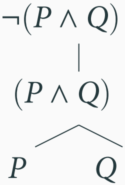

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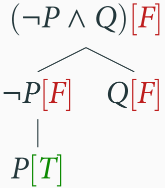

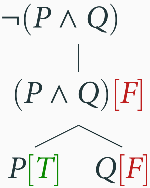

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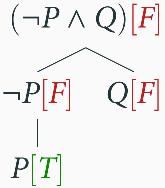

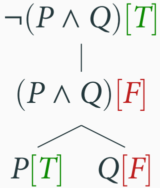

Truth Tables

- Truth-table for '$\neg P \wedge Q$':

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P	Q	$\neg$	P	$\wedge$	Q

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P	Q	$\neg$	$(P$	$\wedge$	$Q)$
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