

Natural Deduction Proofs for SL

Day 3

1		$A \wedge \neg A$	Ass. ($\neg I$)
		—	
2		A	$\wedge E$ 1
3		$\neg A$	$\wedge E$ 1
4		\perp	$\perp I$ 2, 3
5		$\neg(A \wedge \neg A)$	$\neg I$ 1-4

Contradiction Introduction

Contradiction Introduction ($\perp I$)

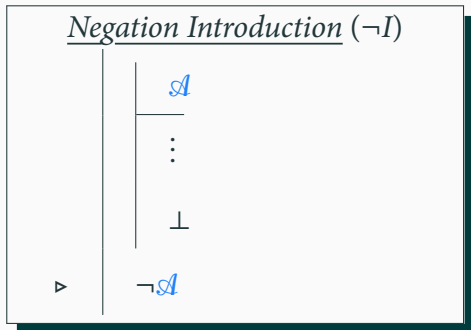
A

$\neg A$

\triangleright

\perp

Negation Introduction



$$P \vdash \neg\neg P$$

$$1 \quad \left| \begin{array}{l} P \end{array} \right.$$

$$P \vdash \neg\neg P$$

$$\begin{array}{l} 1 \\ 2 \end{array} \left| \begin{array}{l} P \\ \hline \neg P \end{array} \right. \text{Ass. } (\neg I)$$

$$P \vdash \neg\neg P$$

1		P	
		—	
2			$\neg P$ Ass. ($\neg I$)
			—
3			\perp $\perp I$ 1, 2

$$P \vdash \neg\neg P$$

1		P	
		—	
2			
2			$\neg P$ Ass. ($\neg I$)
			—
3			\perp $\perp I$ 1, 2
			—
4		$\neg\neg P$	$\neg I$ 2-3

$$P \vdash \neg\neg P$$

1		P	
		—	
2			$\neg P$ Ass. ($\neg I$)
			—
3			\perp $\perp I$ 1, 2
4		$\neg\neg P$	$\neg I$ 2-3

$$X \wedge Y \vdash \neg(\neg Y \wedge X)$$

$$1 \quad \left\{ \begin{array}{l} X \wedge Y \end{array} \right.$$

$$X \wedge Y \vdash \neg(\neg Y \wedge X)$$

$$\begin{array}{l|l} 1 & X \wedge Y \\ \hline 2 & Y \end{array} \quad \wedge E 1$$

$X \wedge Y \vdash \neg(\neg Y \wedge X)$

1		$X \wedge Y$	
		—	
2		Y	$\wedge E\ 1$
3		$\neg Y \wedge X$	Ass. ($\neg I$)

$$X \wedge Y \vdash \neg(\neg Y \wedge X)$$

1		$X \wedge Y$	
		—	
2		Y	$\wedge E$ 1
3		$\neg Y \wedge X$	Ass. ($\neg I$)
		—	
4		$\neg Y$	$\wedge E$ 3
5		\perp	$\perp I$ 2, 4

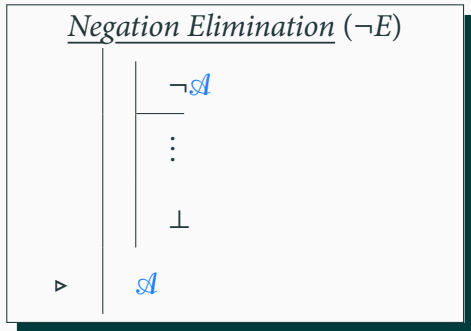
$$X \wedge Y \vdash \neg(\neg Y \wedge X)$$

1		$X \wedge Y$	
		—	
2		Y	$\wedge E$ 1
3			
3			$\neg Y \wedge X$ Ass. ($\neg I$)
			—
4			$\neg Y$ $\wedge E$ 3
5			\perp $\perp I$ 2, 4
6		$\neg(\neg Y \wedge X)$	$\neg I$ 3-5

$X \wedge Y \vdash \neg(\neg Y \wedge X)$

1		$X \wedge Y$	
		—	
2		Y	$\wedge E$ 1
3			
3			$\neg Y \wedge X$ Ass. ($\neg I$)
			—
4			$\neg Y$ $\wedge E$ 3
5			\perp $\perp I$ 2, 4
6		$\neg(\neg Y \wedge X)$	$\neg I$ 3-5

Negation Elimination



$\neg\neg A \vdash A$

1 $\left[\neg\neg A \right.$

$\neg\neg A \vdash A$

1 | $\neg\neg A$
 |_____
2 | | $\neg A$ *Ass. ($\neg E$)*
 | |_____
 |_____
 |

$\neg\neg A \vdash A$

1		$\neg\neg A$	
		—	
2			$\neg A$ Ass. ($\neg E$)
			—
3			\perp $\perp I$ 1, 2

$\neg\neg A \vdash A$

1		$\neg\neg A$	
		—	
2			$\neg A$ <i>Ass. ($\neg E$)</i>
			—
3			\perp $\perp I$ 1, 2
4		A	$\neg E$ 2-3

$\neg\neg A \vdash A$

1		$\neg\neg A$	
		—	
2			$\neg A$ <i>Ass. ($\neg E$)</i>
			—
3			\perp $\perp I$ 1, 2
4		A	$\neg E$ 2-3

$$\vdash G \leftrightarrow \neg\neg G$$

$$1 \quad \left[\begin{array}{l} G \end{array} \right. \quad \text{Ass. } (\leftrightarrow I)$$

$\vdash G \leftrightarrow \neg\neg G$

1		G	Ass. ($\leftrightarrow I$)
		├	
2		├ $\neg G$	Ass. ($\neg I$)
		└	

$\vdash G \leftrightarrow \neg\neg G$

1	G	Ass. ($\leftrightarrow I$)

2	$\neg G$	Ass. ($\neg I$)

3	\perp	$\perp I$ 1, 2

$$\vdash G \leftrightarrow \neg\neg G$$

1		G	Ass. ($\leftrightarrow I$)
2		$\neg G$	Ass. ($\neg I$)
3		\perp	$\perp I$ 1, 2
4		$\neg\neg G$	$\neg I$ 2, 3

$$\vdash G \leftrightarrow \neg\neg G$$

1		G	Ass. ($\leftrightarrow I$)
		—	
2			
2			Ass. ($\neg I$)
			—
3			\perp
			$\perp I$ 1, 2
			—
4			$\neg\neg G$
			$\neg I$ 2, 3
			—
5			$\neg\neg G$
			Ass. ($\leftrightarrow I$)

$\vdash G \leftrightarrow \neg\neg G$

1		G	Ass. ($\leftrightarrow I$)
		—	
2		$\neg G$	Ass. ($\neg I$)
		—	
3		\perp	$\perp I$ 1, 2
4		$\neg\neg G$	$\neg I$ 2, 3
		—	
5		$\neg\neg G$	Ass. ($\leftrightarrow I$)
		—	
6		$\neg G$	Ass. ($\neg E$)
		—	

$\vdash G \leftrightarrow \neg\neg G$

1	G	Ass. ($\leftrightarrow I$)

2	$\neg G$	Ass. ($\neg I$)

3	\perp	$\perp I$ 1, 2
4	$\neg\neg G$	$\neg I$ 2, 3

5	$\neg\neg G$	Ass. ($\leftrightarrow I$)

6	$\neg G$	Ass. ($\neg E$)

7	\perp	$\perp I$ 5, 6

$\vdash G \leftrightarrow \neg\neg G$

1	G	Ass. ($\leftrightarrow I$)
2	$\neg G$	Ass. ($\neg I$)
3	\perp	$\perp I$ 1, 2
4	$\neg\neg G$	$\neg I$ 2, 3
5	$\neg\neg G$	Ass. ($\leftrightarrow I$)
6	$\neg G$	Ass. ($\neg E$)
7	\perp	$\perp I$ 5, 6
8	G	$\neg E$ 6-7

$\vdash G \leftrightarrow \neg\neg G$

1	G	Ass. ($\leftrightarrow I$)
2	$\neg G$	Ass. ($\neg I$)
3	\perp	$\perp I$ 1, 2
4	$\neg\neg G$	$\neg I$ 2, 3
5	$\neg\neg G$	Ass. ($\leftrightarrow I$)
6	$\neg G$	Ass. ($\neg E$)
7	\perp	$\perp I$ 5, 6
8	G	$\neg E$ 6-7
9	$G \leftrightarrow \neg\neg G$	$\leftrightarrow I$ 1-4, 5-8

$\vdash G \leftrightarrow \neg\neg G$

1	G	Ass. ($\leftrightarrow I$)
2	$\neg G$	Ass. ($\neg I$)
3	\perp	$\perp I$ 1, 2
4	$\neg\neg G$	$\neg I$ 2, 3
5	$\neg\neg G$	Ass. ($\leftrightarrow I$)
6	$\neg G$	Ass. ($\neg E$)
7	\perp	$\perp I$ 5, 6
8	G	$\neg E$ 6-7
9	$G \leftrightarrow \neg\neg G$	$\leftrightarrow I$ 1-4, 5-8

Contradiction Elimination

Contradiction Elimination ($\perp E$)

\triangleright

\perp

\mathcal{A}

$$P \rightarrow \neg P \quad \vdash \quad P \rightarrow Q$$

$$1 \quad \left[\begin{array}{l} P \rightarrow \neg P \end{array} \right.$$

$$P \rightarrow \neg P \quad \vdash \quad P \rightarrow Q$$

$$\begin{array}{l} 1 \quad \left| \begin{array}{l} P \rightarrow \neg P \\ \hline \end{array} \right. \\ 2 \quad \left| \begin{array}{l} \left| \begin{array}{l} P \\ \hline \end{array} \right. \end{array} \right. \end{array} \quad \text{Ass. } (\rightarrow I)$$

$P \rightarrow \neg P \quad \vdash \quad P \rightarrow Q$

1		$P \rightarrow \neg P$	
		—	
2			Ass. ($\rightarrow I$)
3			$\neg P$ $\rightarrow E$ 1, 2

$$P \rightarrow \neg P \quad \vdash \quad P \rightarrow Q$$

1		$P \rightarrow \neg P$	
		—	
2			Ass. ($\rightarrow I$)
3			$\neg P$ $\rightarrow E$ 1, 2
4			\perp $\perp I$ 2, 3

$P \rightarrow \neg P \quad \vdash \quad P \rightarrow Q$

1		$P \rightarrow \neg P$	
		—	
2			P Ass. ($\rightarrow I$)
			—
3			$\neg P$ $\rightarrow E$ 1, 2
4			\perp $\perp I$ 2, 3
5			Q $\perp E$ 4

$P \rightarrow \neg P \quad \vdash \quad P \rightarrow Q$

1		$P \rightarrow \neg P$	
		—	
2			P Ass. ($\rightarrow I$)
			—
3			$\neg P$ $\rightarrow E$ 1, 2
4			\perp $\perp I$ 2, 3
5			Q $\perp E$ 4
6		$P \rightarrow Q$	$\rightarrow I$ 2-5

$P \rightarrow \neg P \quad \vdash \quad P \rightarrow Q$

1		$P \rightarrow \neg P$	

2			P <i>Ass. ($\rightarrow I$)</i>

3			$\neg P$ $\rightarrow E$ 1, 2

4			\perp $\perp I$ 2, 3

5			Q $\perp E$ 4

6		$P \rightarrow Q$	$\rightarrow I$ 2-5

Conjunction Introduction

Conjunction Introduction ($\wedge I$)

\mathcal{A}

\mathcal{B}

\triangleright

$\mathcal{A} \wedge \mathcal{B}$

Conjunction Elimination

Conjunction Elimination ($\wedge E$)

$A \wedge B$

▽

A

▽

B

Disjunction Introduction

Disjunction Introduction ($\vee I$)

A

▷

$A \vee B$

▷

$B \vee A$

Disjunction Elimination

Disjunction Elimination ($\vee E$)

$A \vee B$

A

\vdots

C

B

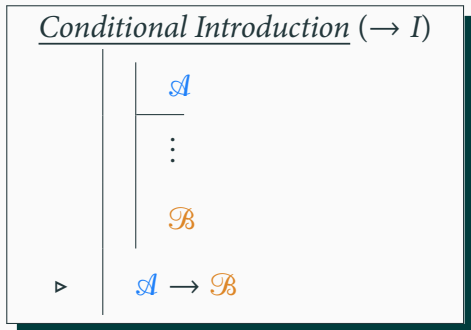
\vdots

C

\triangleright

C

Conditional Introduction



Conditional Elimination

Conditional Elimination ($\rightarrow E$)

$A \rightarrow B$

A

\triangleright

B

Biconditional Introduction

Biconditional Introduction ($\leftrightarrow I$)

	\mathcal{A}
	⋮
	\mathcal{B}
	\mathcal{B}
	⋮
	\mathcal{A}
▷	$\mathcal{A} \leftrightarrow \mathcal{B}$

Biconditional Elimination

Biconditional Elimination ($\leftrightarrow E$)

$$A \leftrightarrow B$$

$$A$$

▽

$$B$$

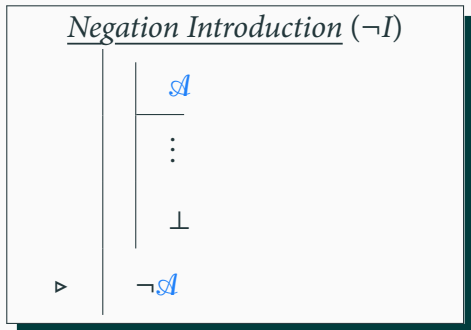
$$A \leftrightarrow B$$

$$B$$

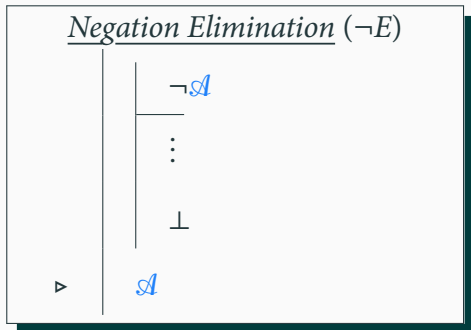
▽

$$A$$

Negation Introduction



Negation Elimination



Contradiction Introduction

Contradiction Introduction ($\perp I$)

A

$\neg A$

\triangleright

\perp

Contradiction Elimination

Contradiction Elimination ($\perp E$)

\triangleright

\perp

\mathcal{A}

Proof Strategies

Proof Strategies

- Constructing a legal natural deduction proof requires *creativity*.

Proof Strategies

- Constructing a legal natural deduction proof requires *creativity*.
- There's no algorithm or step-by-step instructions I can give you.

Proof Strategies

- Constructing a legal natural deduction proof requires *creativity*.
- There's no algorithm or step-by-step instructions I can give you.
- But I have some tips.

Proof Strategies

- Tip #0: come up with a ‘big picture’ strategy

Proof Strategies

- Tip #1: Try to use the introduction rule for the main operator of the sentence you want to write down.

1 | $P \rightarrow \neg P$ Goal: $\neg P$
 | _____
 |

Proof Strategies

- Tip #1: Try to use the introduction rule for the main operator of the sentence you want to write down.

1		$P \rightarrow \neg P$	Goal: $\neg P$

2			Ass. ($\neg I$) Goal: \perp

3			$\neg P$ $\rightarrow E$ 1, 2
			\perp

		$\neg P$	$\neg I$

Proof Strategies

- Tip #1: Try to use the introduction rule for the main operator of the sentence you want to write down.

1		$P \rightarrow \neg P$	Goal: $\neg P$
		—	
2			Ass. ($\neg I$) Goal: \perp
3			$\rightarrow E$ 1, 2
4			$\perp I$ 2, 3
		$\neg P$	$\neg I$

Proof Strategies

- Tip #1: Try to use the introduction rule for the main operator of the sentence you want to write down.

1		$P \rightarrow \neg P$	Goal: $\neg P$
2			Ass. ($\neg I$) Goal: \perp
3			$\rightarrow E$ 1, 2
4			$\perp I$ 2, 3
5		$\neg P$	$\neg I$ 2-5

Proof Strategies

- Tip #1: Try to use the introduction rule for the main operator of the sentence you want to write down.

1		$P \rightarrow \neg P$	Goal: $\neg P$
		—	
2			Ass. ($\neg I$) Goal: \perp
3			$\rightarrow E$ 1, 2
4			$\perp I$ 2, 3
5		$\neg P$	$\neg I$ 2-5

Proof Strategies

- Tip #2: Try to use the elimination rule for the main operator of a sentence you have written down.

Proof Strategies

1		$P \vee Q$		
2		$P \rightarrow Q$	Want: Q	
3			Ass. ($\vee E$)	Goal: ' Q '
4				
5			Ass. ($\vee E$)	Goal: ' Q '
6				
7				
8		Q	$\vee E$	

Proof Strategies

1		$P \vee Q$		
2		$P \rightarrow Q$	Want: Q	
		—		
3			Ass. ($\vee E$)	Goal: ' Q '
4			$\rightarrow E$ 2, 3	
5			Ass. ($\vee E$)	Goal: ' Q '
6				
7				
8		Q	$\vee E$	

Proof Strategies

1		$P \vee Q$		
2		$P \rightarrow Q$	Want: Q	
		—		
3			Ass. ($\vee E$)	Goal: ' Q '
4			$\rightarrow E$ 2, 3	
5			Ass. ($\vee E$)	Goal: ' Q '
6			$\wedge I$ 5	
7				
8		Q	$\vee E$	

Proof Strategies

1		$P \vee Q$		
2		$P \rightarrow Q$	Want: Q	
		—		
3			Ass. ($\vee E$)	Goal: ' Q '
4			$\rightarrow E$ 2, 3	
5			Ass. ($\vee E$)	Goal: ' Q '
6			$\wedge I$ 5	
7			$\wedge E$ 6	
8		Q	$\vee E$	

Proof Strategies

1	$P \vee Q$		
2	$P \rightarrow Q$	Want: Q	
3	P	Ass. ($\vee E$)	Goal: ' Q '
4	Q	$\rightarrow E$ 2, 3	
5	Q	Ass. ($\vee E$)	Goal: ' Q '
6	$Q \wedge Q$	$\wedge I$ 5	
7	Q	$\wedge E$ 6	
8	Q	$\vee E$ 1, 3-4, 5-7	

Proof Strategies

1		$P \vee Q$		
2		$P \rightarrow Q$	Want: Q	
		—		
3			Ass. ($\vee E$)	Goal: 'Q'
4			$\rightarrow E$ 2, 3	
5			Ass. ($\vee E$)	Goal: 'Q'
6			$\wedge I$ 5	
7			$\wedge E$ 6	
8		Q	$\vee E$ 1, 3-4, 5-7	

Proof Strategies

- Tip #3: Form sub-goals.

Proof Strategies

- Tip #3: Form sub-goals.
- Use the sub-goals to structure your thinking about the problem, turning one hard problem into several easier problems.

1

$(A \wedge B) \vee (C \wedge D)$

Goal: $\neg C \rightarrow A$

1	(A ∧ B) ∨ (C ∧ D)		Goal: ¬C → A

2	¬C	Ass. (→ I)	Goal: A

3	A ∧ B	Ass. (∨E)	Goal: A

	C ∧ D	Ass. (∨E)	Goal: A

	A	∨E	

	¬C → A	→ I	

1	(A ∧ B) ∨ (C ∧ D)		Goal: ¬C → A
2	¬C	Ass. (→ I)	Goal: A
3	A ∧ B	Ass. (∨E)	Goal: A
4	A	∧E 3	
5	C ∧ D	Ass. (∨E)	Goal: A
	A	∨E	
	¬C → A	→ I	

1	$(A \wedge B) \vee (C \wedge D)$		Goal: $\neg C \rightarrow A$
2	$\neg C$	Ass. ($\rightarrow I$)	Goal: A
3	$A \wedge B$	Ass. ($\vee E$)	Goal: A
4	A	$\wedge E$ 3	
5	$C \wedge D$	Ass. ($\vee E$)	Goal: A
6	C	$\wedge E$ 5	
	A	$\vee E$	
	$\neg C \rightarrow A$	$\rightarrow I$	

1	$(A \wedge B) \vee (C \wedge D)$		Goal: $\neg C \rightarrow A$
2	$\neg C$	Ass. ($\rightarrow I$)	Goal: A
3	$A \wedge B$	Ass. ($\vee E$)	Goal: A
4	A	$\wedge E$ 3	
5	$C \wedge D$	Ass. ($\vee E$)	Goal: A
6	C	$\wedge E$ 5	
7	\perp	$\perp I$ 2, 6	
	A	$\vee E$	
	$\neg C \rightarrow A$	$\rightarrow I$	

1	$(A \wedge B) \vee (C \wedge D)$		Goal: $\neg C \rightarrow A$
2	<div style="border-left: 1px solid black; padding-left: 10px;">$\neg C$</div>	Ass. ($\rightarrow I$)	Goal: A
3	<div style="border-left: 1px solid black; padding-left: 10px;"> <div style="border-left: 1px solid black; padding-left: 10px;">$A \wedge B$</div> </div>	Ass. ($\vee E$)	Goal: A
4	<div style="border-left: 1px solid black; padding-left: 10px;"> <div style="border-left: 1px solid black; padding-left: 10px;">A</div> </div>	$\wedge E$ 3	
5	<div style="border-left: 1px solid black; padding-left: 10px;"> <div style="border-left: 1px solid black; padding-left: 10px;">$C \wedge D$</div> </div>	Ass. ($\vee E$)	Goal: A
6	<div style="border-left: 1px solid black; padding-left: 10px;"> <div style="border-left: 1px solid black; padding-left: 10px;">C</div> </div>	$\wedge E$ 5	
7	<div style="border-left: 1px solid black; padding-left: 10px;">\perp</div>	$\perp I$ 2, 6	
8	A	$\perp E$ 7	
	A	$\vee E$	
	$\neg C \rightarrow A$	$\rightarrow I$	

1	$(A \wedge B) \vee (C \wedge D)$		Goal: $\neg C \rightarrow A$
2	$\neg C$	Ass. ($\rightarrow I$)	Goal: A
3	$A \wedge B$	Ass. ($\vee E$)	Goal: A
4	A	$\wedge E$ 3	
5	$C \wedge D$	Ass. ($\vee E$)	Goal: A
6	C	$\wedge E$ 5	
7	\perp	$\perp I$ 2, 6	
8	A	$\perp E$ 7	
9	A	$\vee E$ 1, 3-4, 5-8	
	$\neg C \rightarrow A$	$\rightarrow I$	

1	$(A \wedge B) \vee (C \wedge D)$		Goal: $\neg C \rightarrow A$
2	$\neg C$	Ass. ($\rightarrow I$)	Goal: A
3	$A \wedge B$	Ass. ($\vee E$)	Goal: A
4	A	$\wedge E$ 3	
5	$C \wedge D$	Ass. ($\vee E$)	Goal: A
6	C	$\wedge E$ 5	
7	\perp	$\perp I$ 2, 6	
8	A	$\perp E$ 7	
9	A	$\vee E$ 1, 3-4, 5-8	
10	$\neg C \rightarrow A$	$\rightarrow I$ 2-9	

1	(A ∧ B) ∨ (C ∧ D)		Goal: ¬C → A
2	¬C	Ass. (→ I)	Goal: A
3	A ∧ B	Ass. (∨E)	Goal: A
4	A	∧E 3	
5	C ∧ D	Ass. (∨E)	Goal: A
6	C	∧E 5	
7	⊥	⊥I 2, 6	
8	A	⊥E 7	
9	A	∨E 1, 3-4, 5-8	
10	¬C → A	→ I 2-9	

1

$(A \wedge B) \vee (C \wedge D)$

Goal: $\neg C \rightarrow A$

$\neg C \rightarrow A$

1	$(A \wedge B) \vee (C \wedge D)$		Goal: $\neg C \rightarrow A$									
2	<table style="border-collapse: collapse;"> <tr> <td style="border-left: 1px solid black; padding-left: 10px;">$A \wedge B$</td> <td style="padding-left: 10px;">Ass. ($\vee E$)</td> <td style="padding-left: 20px;">Goal: $\neg C \rightarrow A$</td> </tr> <tr> <td style="border-left: 1px solid black; padding-left: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-left: 1px solid black; padding-left: 10px;">$\neg C \rightarrow A$</td> <td></td> <td></td> </tr> </table> </td> <td></td> <td></td> </tr> </table>	$A \wedge B$	Ass. ($\vee E$)	Goal: $\neg C \rightarrow A$	<table style="border-collapse: collapse;"> <tr> <td style="border-left: 1px solid black; padding-left: 10px;">$\neg C \rightarrow A$</td> <td></td> <td></td> </tr> </table>	$\neg C \rightarrow A$						
$A \wedge B$	Ass. ($\vee E$)	Goal: $\neg C \rightarrow A$										
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$\neg C \rightarrow A$												
	$\neg C \rightarrow A$											

1	$(A \wedge B) \vee (C \wedge D)$		Goal: $\neg C \rightarrow A$
2	$A \wedge B$	Ass. ($\vee E$)	Goal: $\neg C \rightarrow A$
	$\neg C \rightarrow A$		
	$C \wedge D$	Ass. ($\vee E$)	Goal: $\neg C \rightarrow A$
	$\neg C \rightarrow A$		
	$\neg C \rightarrow A$		

1	$(A \wedge B) \vee (C \wedge D)$		Goal: $\neg C \rightarrow A$
2	$A \wedge B$	Ass. ($\vee E$)	Goal: $\neg C \rightarrow A$
	$\neg C \rightarrow A$		
	$C \wedge D$	Ass. ($\vee E$)	Goal: $\neg C \rightarrow A$
	$\neg C \rightarrow A$		
	$\neg C \rightarrow A$	$\vee E$	

1	$(A \wedge B) \vee (C \wedge D)$		Goal: $\neg C \rightarrow A$
2	<div style="border-left: 1px solid black; padding-left: 10px;"> $A \wedge B$ </div>	Ass. ($\vee E$)	Goal: $\neg C \rightarrow A$
3	<div style="border-left: 1px solid black; padding-left: 10px;"> <div style="border-left: 1px solid black; padding-left: 10px;"> $\neg C$ </div> </div>	Ass. ($\rightarrow I$)	Goal: A
	<div style="border-left: 1px solid black; padding-left: 10px;"> $\neg C \rightarrow A$ </div>		
	<div style="border-left: 1px solid black; padding-left: 10px;"> $C \wedge D$ </div>	Ass. ($\vee E$)	Goal: $\neg C \rightarrow A$
	<div style="border-left: 1px solid black; padding-left: 10px;"> $\neg C \rightarrow A$ </div>		
	$\neg C \rightarrow A$	$\vee E$	

1	$(A \wedge B) \vee (C \wedge D)$		
2	$A \wedge B$	Ass. ($\vee E$)	Goal: $\neg C \rightarrow A$
3	$\neg C$	Ass. ($\rightarrow I$)	Goal: A
4	A	$\wedge E$ 2	
	$\neg C \rightarrow A$		
	$C \wedge D$	Ass. ($\vee E$)	Goal: $\neg C \rightarrow A$
	$\neg C \rightarrow A$		
	$\neg C \rightarrow A$	$\vee E$	

1	$(A \wedge B) \vee (C \wedge D)$		Goal: $\neg C \rightarrow A$
2	$A \wedge B$	Ass. ($\vee E$)	Goal: $\neg C \rightarrow A$
3	$\neg C$	Ass. ($\rightarrow I$)	Goal: A
4	A	$\wedge E$ 2	
5	$\neg C \rightarrow A$	$\rightarrow I$ 3-4	
6	$C \wedge D$	Ass. ($\vee E$)	Goal: $\neg C \rightarrow A$
	$\neg C \rightarrow A$		
	$\neg C \rightarrow A$	$\vee E$	

1	$(A \wedge B) \vee (C \wedge D)$		Goal: $\neg C \rightarrow A$
2	$A \wedge B$	Ass. ($\vee E$)	Goal: $\neg C \rightarrow A$
3	$\neg C$	Ass. ($\rightarrow I$)	Goal: A
4	A	$\wedge E$ 2	
5	$\neg C \rightarrow A$	$\rightarrow I$ 3-4	
6	$C \wedge D$	Ass. ($\vee E$)	Goal: $\neg C \rightarrow A$
7	$\neg C$	Ass. ($\rightarrow I$)	Goal: A
	A		
	$\neg C \rightarrow A$	$\rightarrow I$	
	$\neg C \rightarrow A$	$\vee E$	

1	$(A \wedge B) \vee (C \wedge D)$		Goal: $\neg C \rightarrow A$
2	$A \wedge B$	Ass. ($\vee E$)	Goal: $\neg C \rightarrow A$
3	$\neg C$	Ass. ($\rightarrow I$)	Goal: A
4	A	$\wedge E$ 2	
5	$\neg C \rightarrow A$	$\rightarrow I$ 3-4	
6	$C \wedge D$	Ass. ($\vee E$)	Goal: $\neg C \rightarrow A$
7	$\neg C$	Ass. ($\rightarrow I$)	Goal: A
8	C	$\wedge E$ 6	
	A		
	$\neg C \rightarrow A$	$\rightarrow I$	
	$\neg C \rightarrow A$	$\vee E$	

1	$(A \wedge B) \vee (C \wedge D)$		Goal: $\neg C \rightarrow A$
2	$A \wedge B$	Ass. ($\vee E$)	Goal: $\neg C \rightarrow A$
3	$\neg C$	Ass. ($\rightarrow I$)	Goal: A
4	A	$\wedge E$ 2	
5	$\neg C \rightarrow A$	$\rightarrow I$ 3-4	
6	$C \wedge D$	Ass. ($\vee E$)	Goal: $\neg C \rightarrow A$
7	$\neg C$	Ass. ($\rightarrow I$)	Goal: A
8	C	$\wedge E$ 6	
9	\perp	$\perp I$ 7, 8	
	A		
	$\neg C \rightarrow A$	$\rightarrow I$	
	$\neg C \rightarrow A$	$\vee E$	

1	$(A \wedge B) \vee (C \wedge D)$		Goal: $\neg C \rightarrow A$
2	$A \wedge B$	Ass. ($\vee E$)	Goal: $\neg C \rightarrow A$
3	$\neg C$	Ass. ($\rightarrow I$)	Goal: A
4	A	$\wedge E$ 2	
5	$\neg C \rightarrow A$	$\rightarrow I$ 3-4	
6	$C \wedge D$	Ass. ($\vee E$)	Goal: $\neg C \rightarrow A$
7	$\neg C$	Ass. ($\rightarrow I$)	Goal: A
8	C	$\wedge E$ 6	
9	\perp	$\perp I$ 7, 8	
10	A	$\perp E$ 9	
	$\neg C \rightarrow A$	$\rightarrow I$	
	$\neg C \rightarrow A$	$\vee E$	

1	$(A \wedge B) \vee (C \wedge D)$		Goal: $\neg C \rightarrow A$
2	$A \wedge B$	Ass. ($\vee E$)	Goal: $\neg C \rightarrow A$
3	$\neg C$	Ass. ($\rightarrow I$)	Goal: A
4	A	$\wedge E$ 2	
5	$\neg C \rightarrow A$	$\rightarrow I$ 3-4	
6	$C \wedge D$	Ass. ($\vee E$)	Goal: $\neg C \rightarrow A$
7	$\neg C$	Ass. ($\rightarrow I$)	Goal: A
8	C	$\wedge E$ 6	
9	\perp	$\perp I$ 7, 8	
10	A	$\perp E$ 9	
11	$\neg C \rightarrow A$	$\rightarrow I$ 7-10	
	$\neg C \rightarrow A$	$\vee E$	

1	(A ∧ B) ∨ (C ∧ D)		Goal: ¬C → A
2	A ∧ B	Ass. (∨E)	Goal: ¬C → A
3	¬C	Ass. (→ I)	Goal: A
4	A	∧E 2	
5	¬C → A	→ I 3-4	
6	C ∧ D	Ass. (∨E)	Goal: ¬C → A
7	¬C	Ass. (→ I)	Goal: A
8	C	∧E 6	
9	⊥	⊥I 7, 8	
10	A	⊥E 9	
11	¬C → A	→ I 7-10	
12	¬C → A	∨E 1, 2-5, 6-11	

Proof Strategies

- Tip #4: Keep in mind: once you have a contradiction, \perp , you can get *anything you want*

$$\begin{array}{c|c} & \vdots \\ \hline n & \perp \end{array}$$

Proof Strategies

- Tip #4: Keep in mind: once you have a contradiction, \perp , you can get *anything you want*

		\vdots	
n		\perp	
n+1		\mathcal{A}	$\perp E, n$

Proof Strategies

- Tip #5: Think about meaning.

Proof Strategies

- Tip #5: Think about meaning.
- ▶ If a strategy requires you to derive \mathcal{B} from \mathcal{A} , first consider whether \mathcal{B} *follows from* \mathcal{A} . If not, abandon that strategy.

$\neg(A \wedge B) \quad \vdash \quad \neg A \vee \neg B$

1

$\neg(A \wedge B)$

Goal: $\neg A \vee \neg B$

$\neg(A \wedge B) \quad \vdash \quad \neg A \vee \neg B$

1 | $\neg(A \wedge B)$

 | A

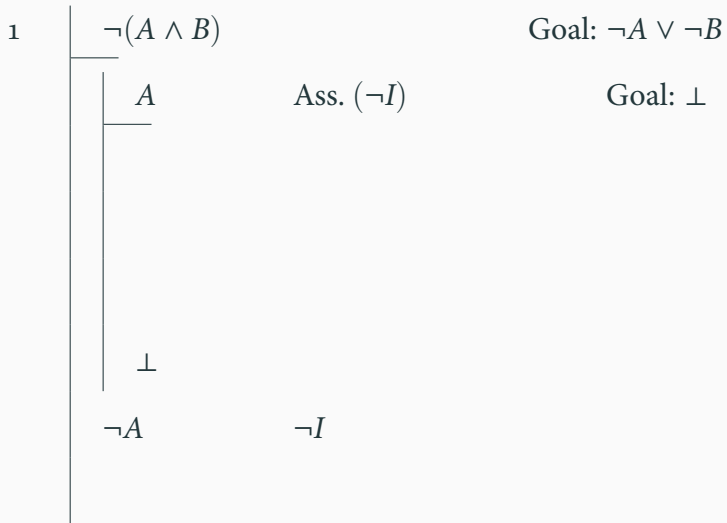
 | Ass. ($\neg I$)

 | \perp

Goal: $\neg A \vee \neg B$

Goal: \perp

$\neg(A \wedge B) \quad \vdash \quad \neg A \vee \neg B$



Proof Strategies

- Tip #6: If you see no other strategies, try negation elimination

Proof Strategies

- Tip #6: If you see no other strategies, try negation elimination
- If anything works, negation elimination will.

$$\neg(A \wedge B) \quad \vdash \quad \neg A \vee \neg B$$

1 | $\neg(A \wedge B)$

Goal: $\neg A \vee \neg B$

$\neg(A \wedge B) \quad \vdash \quad \neg A \vee \neg B$

1		$\neg(A \wedge B)$		Goal: $\neg A \vee \neg B$
		—		
			$\neg(\neg A \vee \neg B)$ Ass. ($\neg E$)	Goal: \perp
			—	
			\perp	
			—	
		$\neg A \vee \neg B$	$\neg E$	

- Tip #7: If you don't have any ideas, *just do something*

1 | $\neg(A \wedge B)$
2 | $\neg(\neg A \vee \neg B)$ Ass. ($\neg E$)

Goal: $\neg A \vee \neg B$

Goal: \perp

1	<div style="border-bottom: 1px solid black; display: inline-block; width: 100px;"></div> $\neg(A \wedge B)$	
2	<div style="border-left: 1px solid black; padding-left: 5px;"> <div style="border-bottom: 1px solid black; display: inline-block; width: 80px;"></div> $\neg(\neg A \vee \neg B)$ </div>	Ass. ($\neg E$)
3	<div style="border-left: 1px solid black; padding-left: 5px;"> <div style="border-left: 1px solid black; padding-left: 5px;"> <div style="border-bottom: 1px solid black; display: inline-block; width: 40px;"></div> A </div> </div>	

Goal: $\neg A \vee \neg B$

Goal: \perp

1	$\neg(A \wedge B)$	
2	$\neg(\neg A \vee \neg B)$	Ass. ($\neg E$)
3	A	Ass. (???)
4	B	Ass. (???)

Goal: $\neg A \vee \neg B$

Goal: \perp

1		$\neg(A \wedge B)$			
		├			
2			$\neg(\neg A \vee \neg B)$ Ass. ($\neg E$)		
			├		
3				A Ass. (???)	
				├	
4					B Ass. (???)
					├
5					$A \wedge B$ $\wedge I$ 3, 4

Goal: $\neg A \vee \neg B$

Goal: \perp

1		$\neg(A \wedge B)$	
2			
3			
4			
5			
6			

		$\neg(\neg A \vee \neg B)$	Ass. ($\neg E$)
		A	Ass. (???)
		B	Ass. (???)
		$A \wedge B$	$\wedge I$ 3, 4
		\perp	$\perp I$ 1, 5

Goal: $\neg A \vee \neg B$

Goal: \perp

1		$\neg(A \wedge B)$	
2			
2			$\neg(\neg A \vee \neg B)$ Ass. ($\neg E$)
3			
3			
3			A Ass. (???)
4			
4			
4			B Ass. ($\neg I$)
5			
5			$A \wedge B$ $\wedge I$ 3, 4
6			
6			\perp $\perp I$ 1, 5
7			$\neg B$ $\neg I$ 4-6

Goal: $\neg A \vee \neg B$

Goal: \perp

1	$\neg(A \wedge B)$	
2	$\neg(\neg A \vee \neg B)$	Ass. ($\neg E$)
3	A	Ass. (???)
4	B	Ass. ($\neg I$)
5	$A \wedge B$	$\wedge I$ 3, 4
6	\perp	$\perp I$ 1, 5
7	$\neg B$	$\neg I$ 4-6
8	$\neg A \vee \neg B$	$\vee I$ 7

Goal: $\neg A \vee \neg B$

Goal: \perp

1		$\neg(A \wedge B)$	
2			
3			
4			
5			
6			
7			
8			
9			

		$\neg(\neg A \vee \neg B)$	Ass. ($\neg E$)
		A	Ass. (???)
		B	Ass. ($\neg I$)
		$A \wedge B$	$\wedge I$ 3, 4
		\perp	$\perp I$ 1, 5
		$\neg B$	$\neg I$ 4-6
		$\neg A \vee \neg B$	$\vee I$ 7
		\perp	$\perp I$ 2, 8

Goal: $\neg A \vee \neg B$

Goal: \perp

1	$\neg(A \wedge B)$	
2	$\neg(\neg A \vee \neg B)$	Ass. ($\neg E$)
3	A	Ass. ($\neg I$)
4	B	Ass. ($\neg I$)
5	$A \wedge B$	$\wedge I$ 3, 4
6	\perp	$\perp I$ 1, 5
7	$\neg B$	$\neg I$ 4-6
8	$\neg A \vee \neg B$	$\vee I$ 7
9	\perp	$\perp I$ 2, 8

Goal: $\neg A \vee \neg B$

Goal: \perp

1		$\neg(A \wedge B)$	
2			
2			$\neg(\neg A \vee \neg B)$ <i>Ass. ($\neg E$)</i>
3			
3			
3			A <i>Ass. ($\neg I$)</i>
4			
4			
4			B <i>Ass. ($\neg I$)</i>
5			
5			$A \wedge B$ $\wedge I$ 3, 4
6			
6			\perp $\perp I$ 1, 5
7			$\neg B$ $\neg I$ 4-6
8			$\neg A \vee \neg B$ $\vee I$ 7
9			\perp $\perp I$ 2, 8
10		$\neg A$ $\neg I$ 3-9	

Goal: $\neg A \vee \neg B$

Goal: \perp

1		$\neg(A \wedge B)$	
2			
3			
4			
5			
6			
7			
8			
9			
10		$\neg A$	
11		$\neg A \vee \neg B$	

Goal: $\neg A \vee \neg B$

Goal: \perp

Ass. ($\neg E$)

Ass. ($\neg I$)

Ass. ($\neg I$)

$\wedge I$ 3, 4

$\perp I$ 1, 5

$\neg I$ 4-6

$\vee I$ 7

$\perp I$ 2, 8

$\neg I$ 3-9

$\vee I$ 10

1		$\neg(A \wedge B)$	
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			

Goal: $\neg A \vee \neg B$

Goal: \perp

$\neg(\neg A \vee \neg B)$ Ass. ($\neg E$)

A Ass. ($\neg I$)

B Ass. ($\neg I$)

$A \wedge B$ $\wedge I$ 3, 4

\perp $\perp I$ 1, 5

$\neg B$ $\neg I$ 4-6

$\neg A \vee \neg B$ $\vee I$ 7

\perp $\perp I$ 2, 8

$\neg A$ $\neg I$ 3-9

$\neg A \vee \neg B$ $\vee I$ 10

\perp $\perp I$ 2, 11

1	$\neg(A \wedge B)$	
2	$\neg(\neg A \vee \neg B)$	Ass. ($\neg E$)
3	A	Ass. ($\neg I$)
4	B	Ass. ($\neg I$)
5	$A \wedge B$	$\wedge I$ 3, 4
6	\perp	$\perp I$ 1, 5
7	$\neg B$	$\neg I$ 4-6
8	$\neg A \vee \neg B$	$\vee I$ 7
9	\perp	$\perp I$ 2, 8
10	$\neg A$	$\neg I$ 3-9
11	$\neg A \vee \neg B$	$\vee I$ 10
12	\perp	$\perp I$ 2, 11
13	$\neg A \vee \neg B$	$\neg E$ 2-12

Goal: $\neg A \vee \neg B$

Goal: \perp